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engineering

Welcome

Engineering Insights 2008

Reduced Order Models for Airflows in Buildings

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Outline

- **Design of Naturally Cool Buildings: A Lost Art?**
- **Proper Orthogonal Decomposition and Galerkin Projection**
- **Academic Example: Plane Couette Flow**
- **Industrial Example: High Performance Buildings**
- **Looking Forward**

Design of Naturally Cool Buildings: A Lost Art?

Historically, design techniques were used so that a building was naturally cool:

- **Barriers against entry of heat**
- **Massive masonry shells that absorb heat (thermal capacitors), perhaps "drain" heat to open body of water**
- **Use of "cool material", such as marble**

Cool Buildings: Some New Ideas

Chicago City Hall roof garden: 20,000 square feet

Can keep roof as much as 70 degrees cooler,
so less energy needed to keep building cool

Cool Buildings: Some New Ideas

Zuidas section of Amsterdam

Deep water from local man-made lake used to cool another supply of water for “cold-radiators” in buildings

Provides air-conditioning for 700,000 people

Some New Ideas: Control of Airflows

- **Goal:**

  the design and active control of indoor airflows to improve ventilation and efficiency of heating and cooling in buildings

- **Approach:**

  develop low-dimensional models which capture enough physics to be meaningful, but which are computationally inexpensive and allow the development of control strategies

Proper Orthogonal Decomposition

- Basic Theory
  - consider set of velocity snapshots $\{\mathrm{u}(\mathrm{x})\}$
  - find basis of POD modes $\{\Phi\}$ which maximizes $\frac{\langle|(\mathrm{u},\Phi)|^2\rangle}{||\Phi||^2}$

- Properties of POD Modes
  - orthogonality (normalization $\rightarrow$ orthonormality)
  - POD modes individually satisfy incompressibility, BCs
  - optimality: for a given number of modes energy captured by POD basis $>$ any other basis

Galerkin Projection

- write evolution PDE as

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{F}(\mathbf{u})$$

- expand

$$\mathbf{u}(\mathbf{x}, t) = \sum_n a_n(t)\mathbf{\Phi}_n(\mathbf{x}),$$

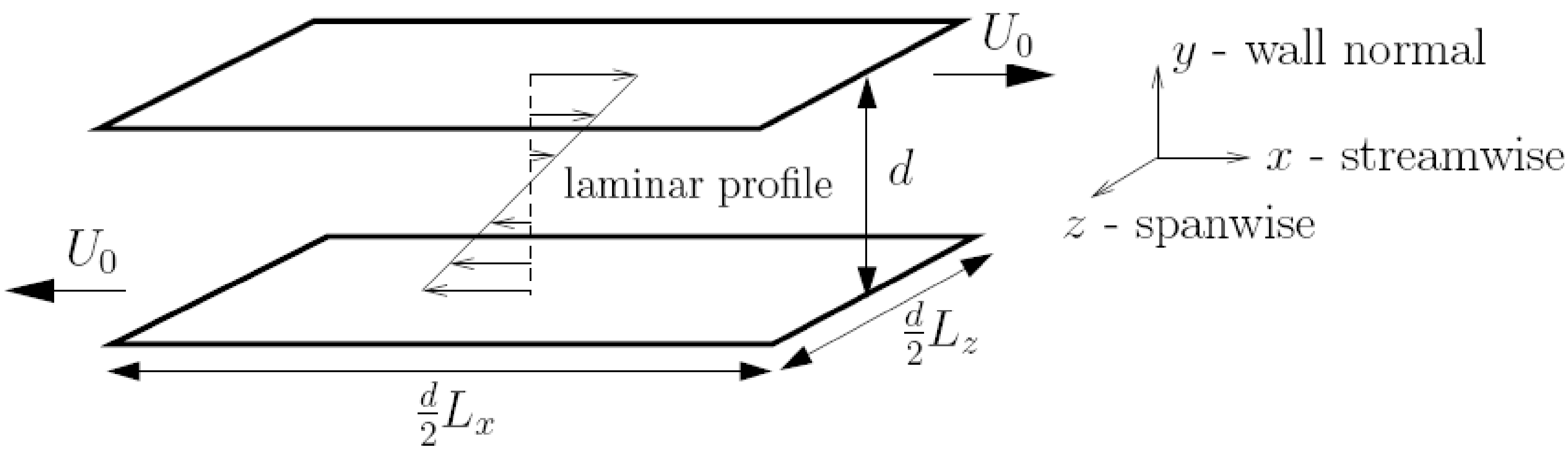
- substitute into evolution PDE

$$\sum_n \dot{a}_n(t)\mathbf{\Phi}_n(\mathbf{x}) = \mathbf{F}\left(\sum_n a_n(t)\mathbf{\Phi}_n(\mathbf{x})\right)$$

- take inner product with $\mathbf{\Phi}_m(\mathbf{x})$, use orthonormality

$$\Rightarrow \dot{a}_m(t) = \left(\mathbf{F}\left(\sum_n a_n(t)\mathbf{\Phi}_n(\mathbf{x})\right), \mathbf{\Phi}_m(\mathbf{x})\right)$$

Academic Example: Plane Couette Flow



Nondimensional equations for perturbation to laminar state:

$$\frac{\partial}{\partial t}\mathbf{u} = -(\mathbf{u}\cdot\nabla)\mathbf{u} - y\frac{\partial}{\partial x}\mathbf{u} - v\,\mathbf{e}_x - \nabla p + \frac{1}{Re}\nabla^2\mathbf{u}$$

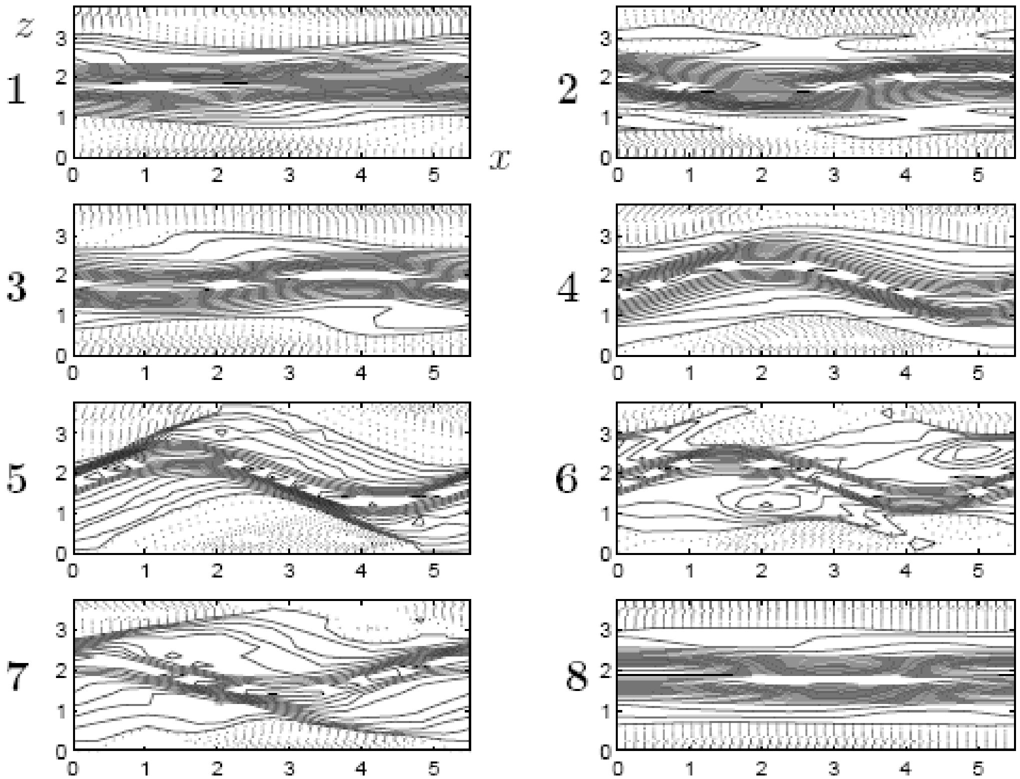
$$\nabla\cdot\mathbf{u} = 0 \qquad \mathbf{u}|_{y=\pm 1} = 0 \qquad Re = \frac{U_0(d/2)}{\nu}$$

Periodic Boundary Conditions in x and z

Minimal Flow Unit: $L_x = 1.75\,\pi, L_z = 1.2\,\pi$

- Streak Breakdown

Midplane x-velocity contours



Minimal Flow Unit: $L_x = 1.75\,\pi, L_z = 1.2\,\pi$

- POD decomposition of PCF–MFU

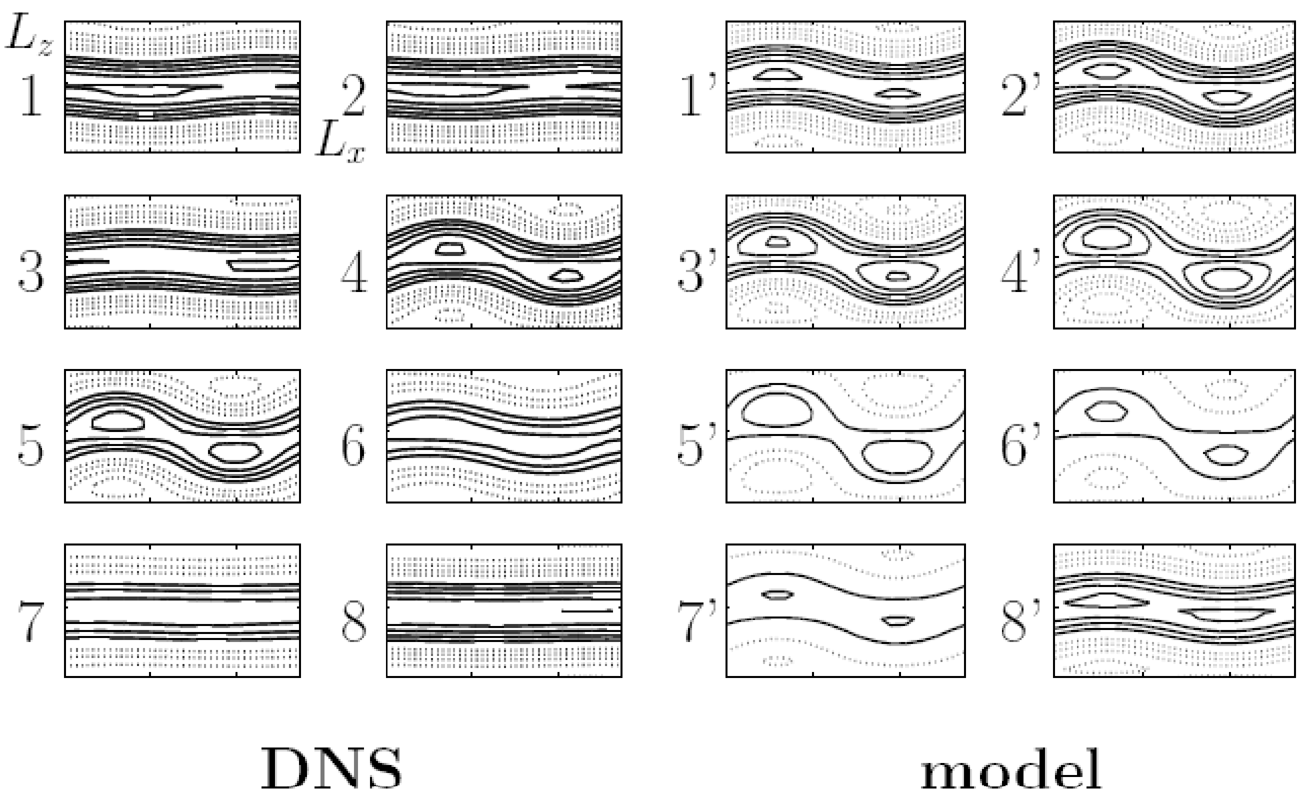
(n, n_x, n_z)	$\lambda^{(n)}_{n_x n_z}$	$\%E^{(n)}_{n_x n_z}$	Model
$(1, 0, 0)$	4.4550	68.02	■
$(1, 0, \pm 1)$	0.7821	23.88	■
$(1, 0, \pm 2)$	0.0543	1.66	■
$(1, \pm 1, 0)$	0.0386	1.18	■
$(1, 0, \pm 3)$	0.0195	0.59	"more of same"
$(2, 0, 0)$	0.0174	0.27	"exclude 2nd family"
$(2, 0, \pm 1)$	0.0123	0.38	"exclude 2nd family"
$(1, \pm 1, \pm 2)$	0.0109	0.33	"Fourier interactions"
$(1, \pm 1, \pm 1)$	0.0090	0.27	■
...			

- 11-dimensional dynamical system

Model includes terms for modes not in truncation

Minimal Flow Unit: $L_x = 1.75\,\pi, L_z = 1.2\,\pi$

- comparison of DNS and model
  - midplane x-velocity contours



Summary of Procedure

snapshots for PCF turbulence (DNS)

↓ eigenvalue problem

POD modes

↓ Galerkin projection, truncation

ODE models

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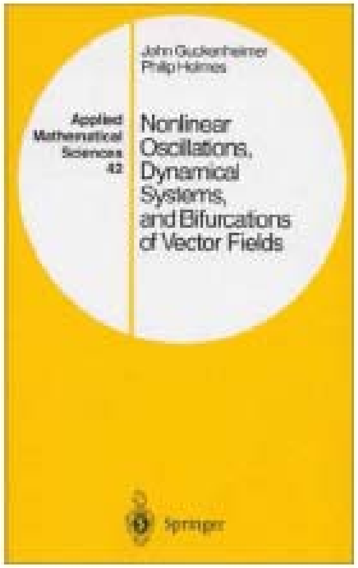
insight into turbulence



Low-dimensional model ⟶ Control

Industrial Example:
High Performance Buildings

Slides: adapted from Amit Surana (United Technologies Research Center)

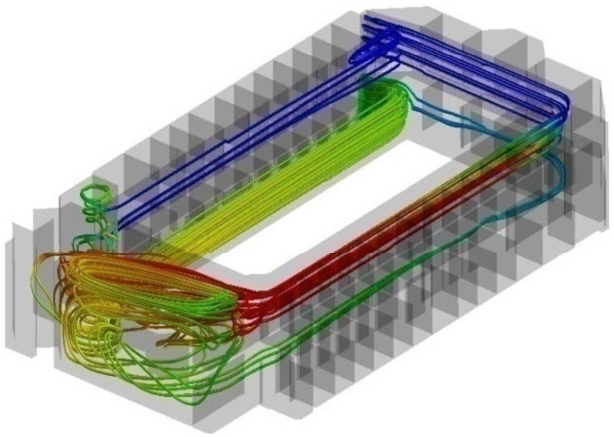
Normal Building Operations:

Optimize comfort, indoor air quality, energy efficiency

Emergency Operations:

Real-time information on contaminant/smoke propagation
Support first-responders, & evacuation management

Flow structures in large spaces have significant effects on transport : thermal, smoke & contaminants



High Performance Buildings

Role of Coherent Structures in

1) Mixing dynamics
2) Strategies for sensing & actuation
3) Reduced order modeling
4) Feedback control of mixing

High Performance Buildings

Airflow Dynamics
Boussinesq approximation,
negligible viscous dissipation

$$\begin{aligned} \nabla \cdot \mathbf{v} &= 0, \\ \rho_0 \left(\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x_i} \right) &= -\frac{\partial p'}{\partial x_i} - \rho_0 \beta T_d g_i + \mu \nabla^2 \mathbf{v}, \\ \rho_0 C_p \left(\frac{\partial T_d}{\partial t} + v_i \frac{\partial T_d}{\partial x_i} \right) &= k \nabla^2 T_d, \end{aligned}$$

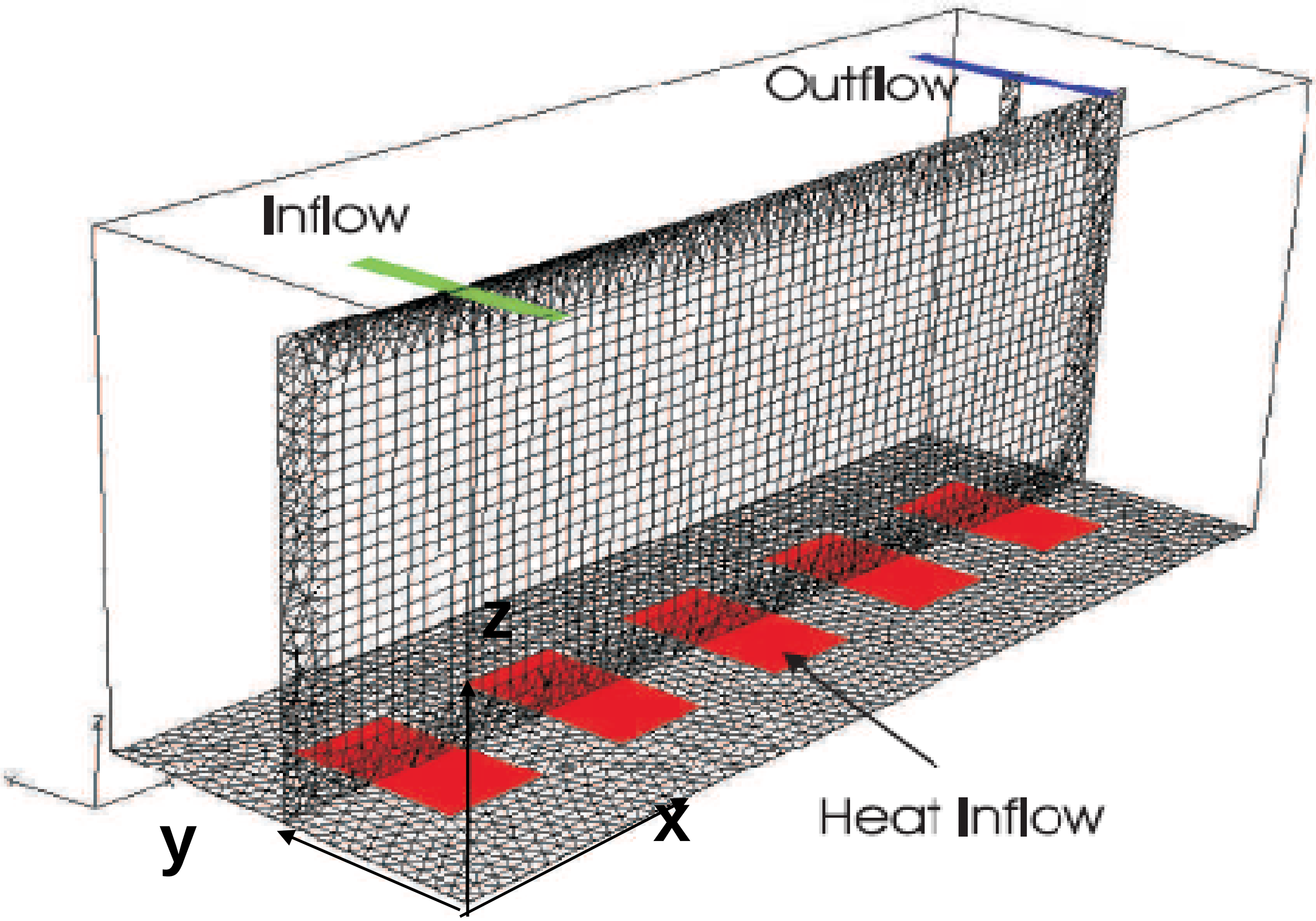
Expansion in POD Models

$$\begin{aligned} \mathbf{v}(\mathbf{x}, t) &= \mathbf{v}^0(\mathbf{x}) + \sum_{i=1}^{n} a_i(t) \phi_i(\mathbf{x}), \\ T(\mathbf{x}, t) &= T^0(\mathbf{x}) + \sum_{i=1}^{p} b_i(t) \psi_i(\mathbf{x}). \end{aligned}$$

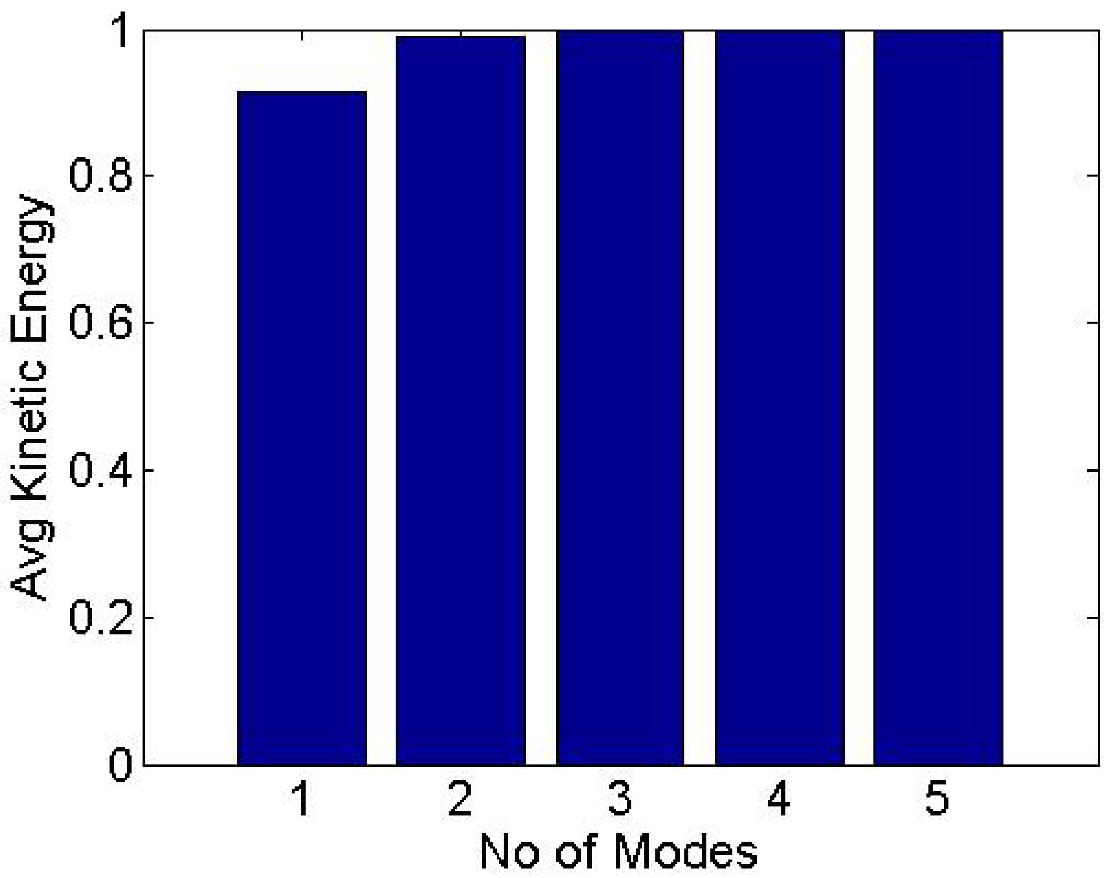
Galerkin Projection $\longrightarrow$ **Low-Dimensional Model**

Initial Conditions? Sensors

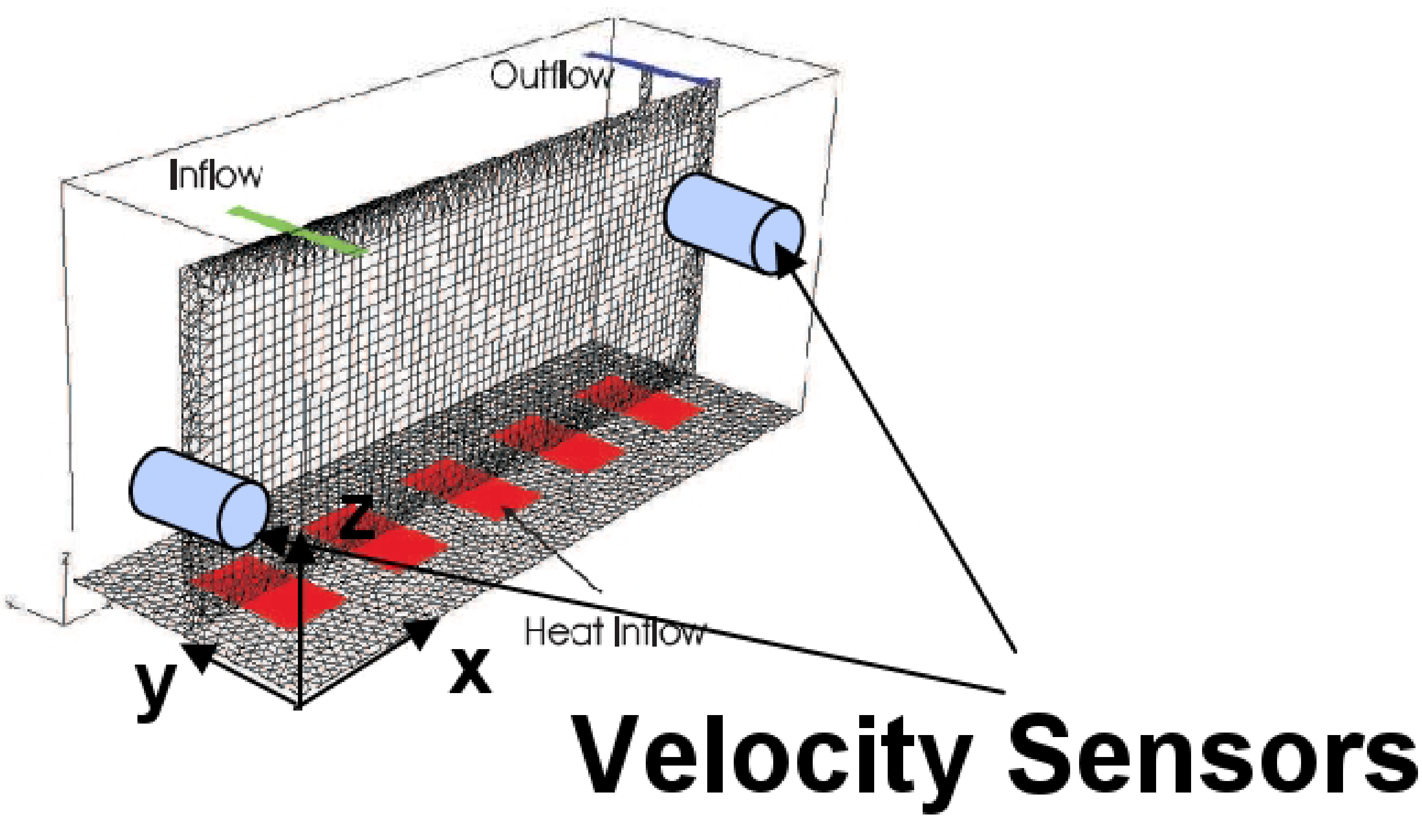
Model Reduction for 3D Room



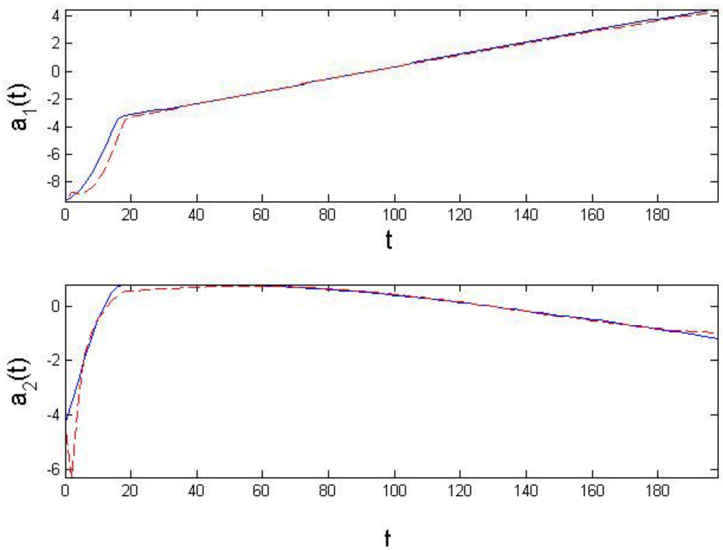
Model Reduction for 3D Room



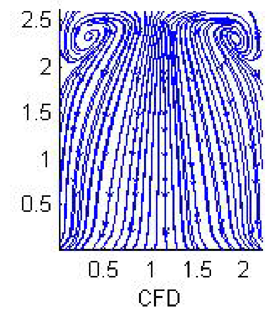
Model Reduction for 3D Room



Model Reduction for 3D Room

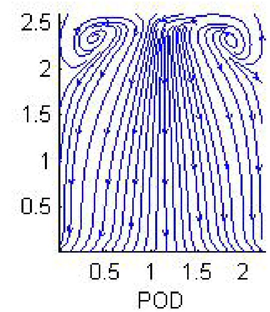


DNS

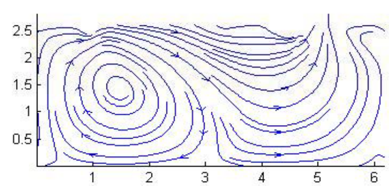


X-Slice

model

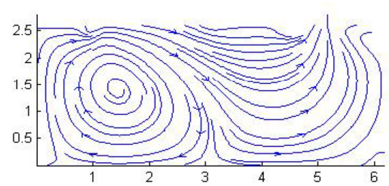


DNS

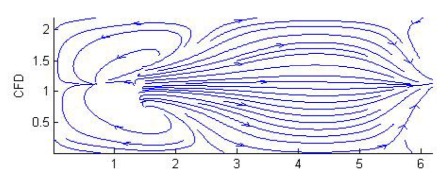


Y-Slice

model

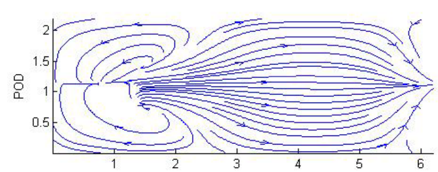


DNS



Z-Slice

model

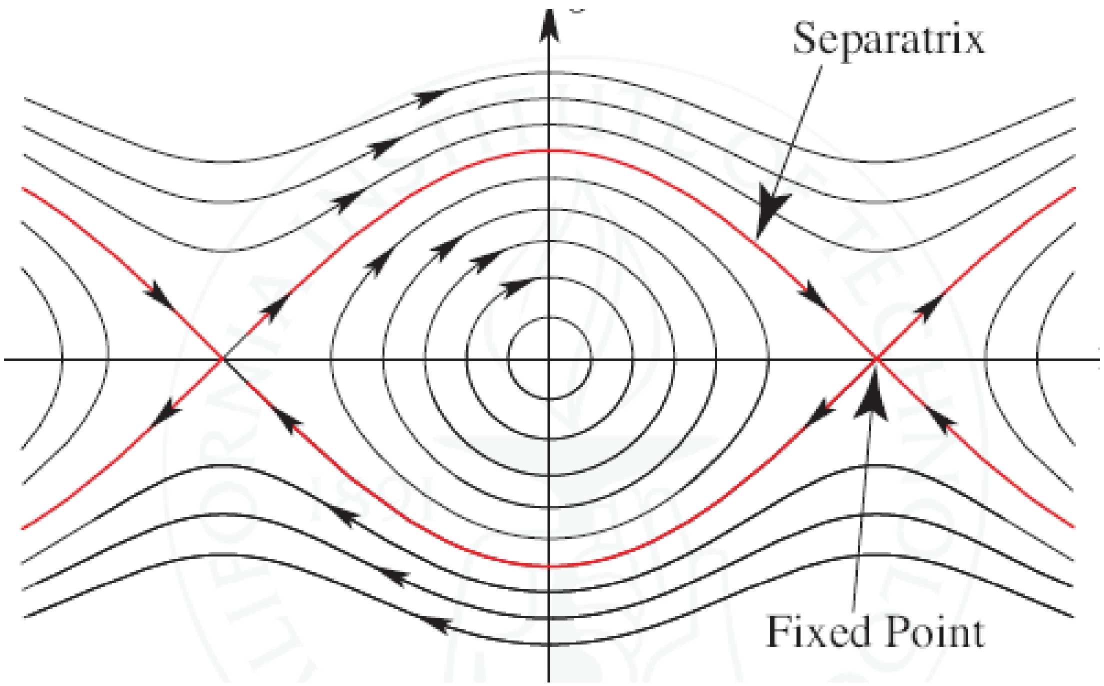


UTRC Proprietary

Lagrangian Coherent Structures

Underlying structures that dictate transport and mixing

Straining Regions: stretching, folding & alignment
Vortices: trap material

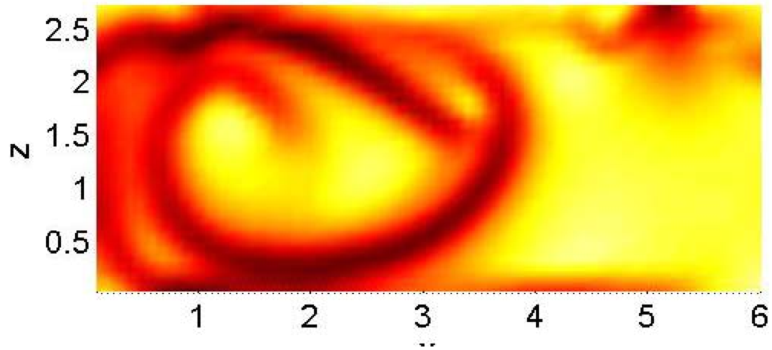


Thanks to Jerry Marsden, Caltech

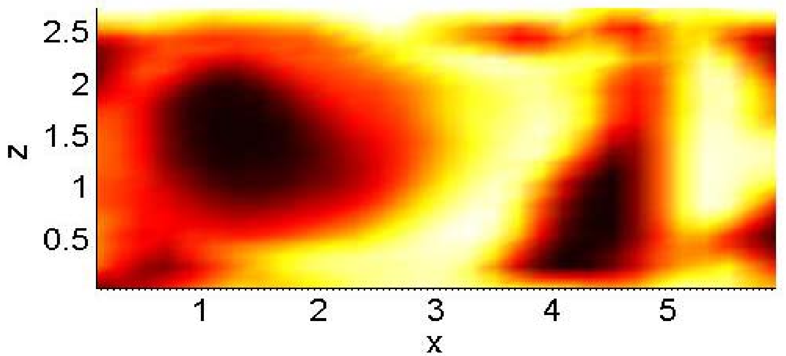
Application to Building Environment

- **Mechanical Ventilation**

DLE



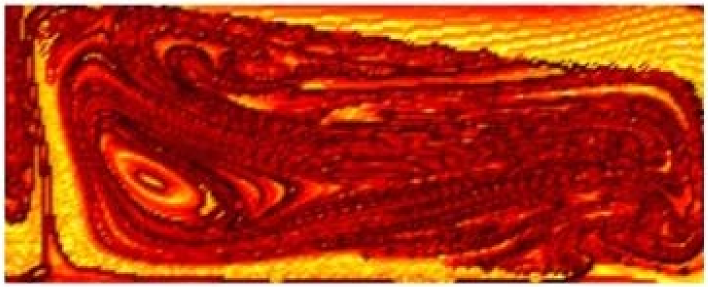
M_Z



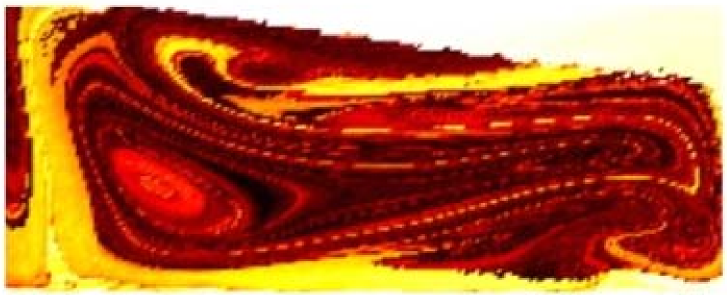
Application to Building Environment

- **Mechanical Ventilation & Moderate Heating**

DLE



M_Z



Application to Building Environment

- **Mechanical Ventilation & Continuous Heating**

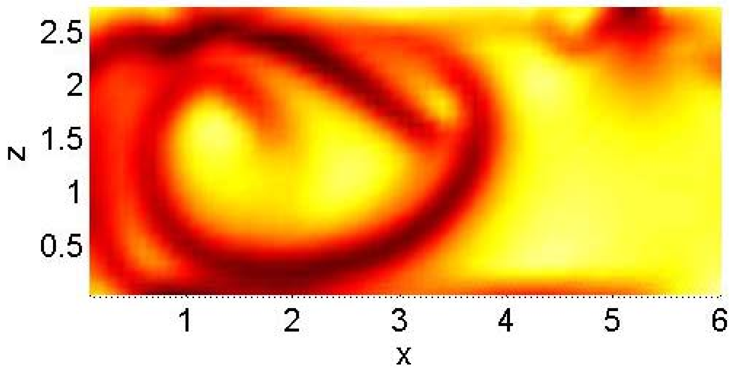
DLE

M_Z

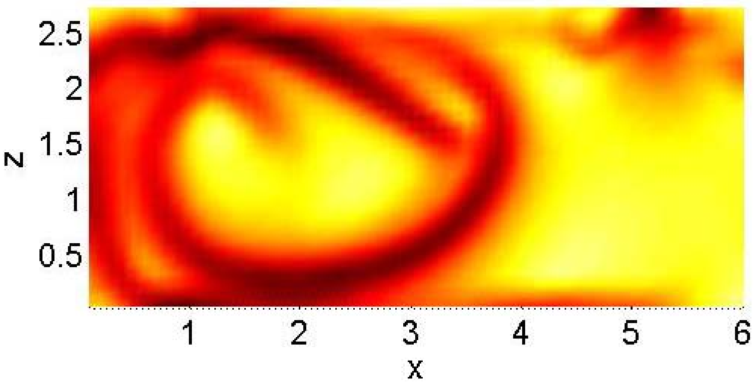
UTRC Proprietary

LCS Based Model Reduction

DNS



1 Mode



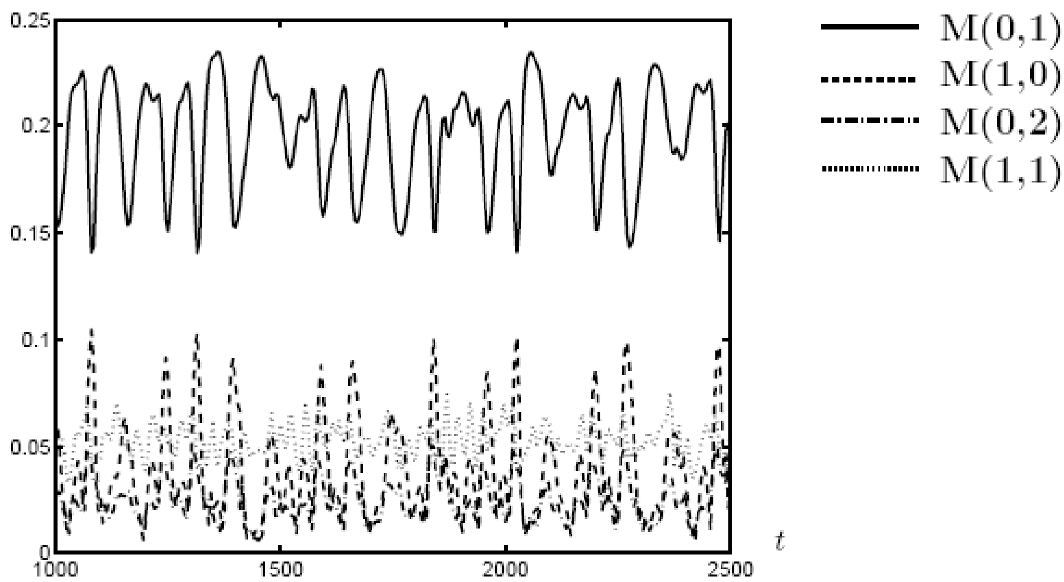
1 Mode is sufficient to capture transport

Looking Forward

- **Better low-dimensional models:**
  - **- modeling loses to truncated modes**
  - **- models which minimize error over specified time interval**
  - **- modes based more directly on Lagrangian Coherent Structures**
  - **- treatment of multiple rooms connected by vents/hallways**
- **Optimal sensor placement and estimation**
- **Optimal control of mixing**

Minimal Flow Unit: $L_x = 1.75\,\pi, L_z = 1.2\,\pi$

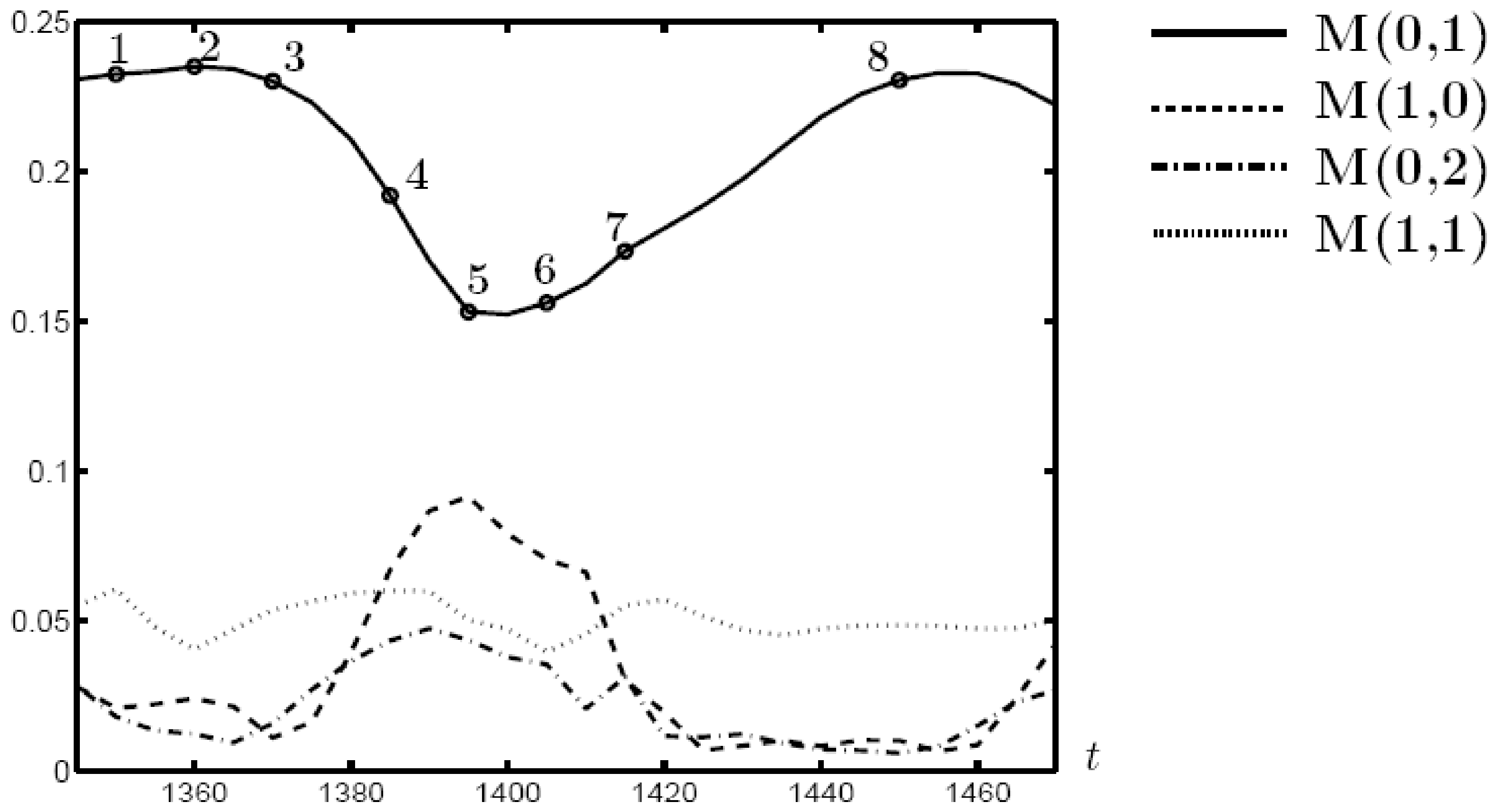
- MFU: smallest domain in which turbulence can be sustained - Hamilton, Kim, Waleffe *JFM* 1995
- define modal RMS velocity $\mathrm{M}(n_x, n_z)$ as energy contained in wavenumbers n_x, n_z; for $Re = 400$



- flow is roughly periodic with period $80 - 100$

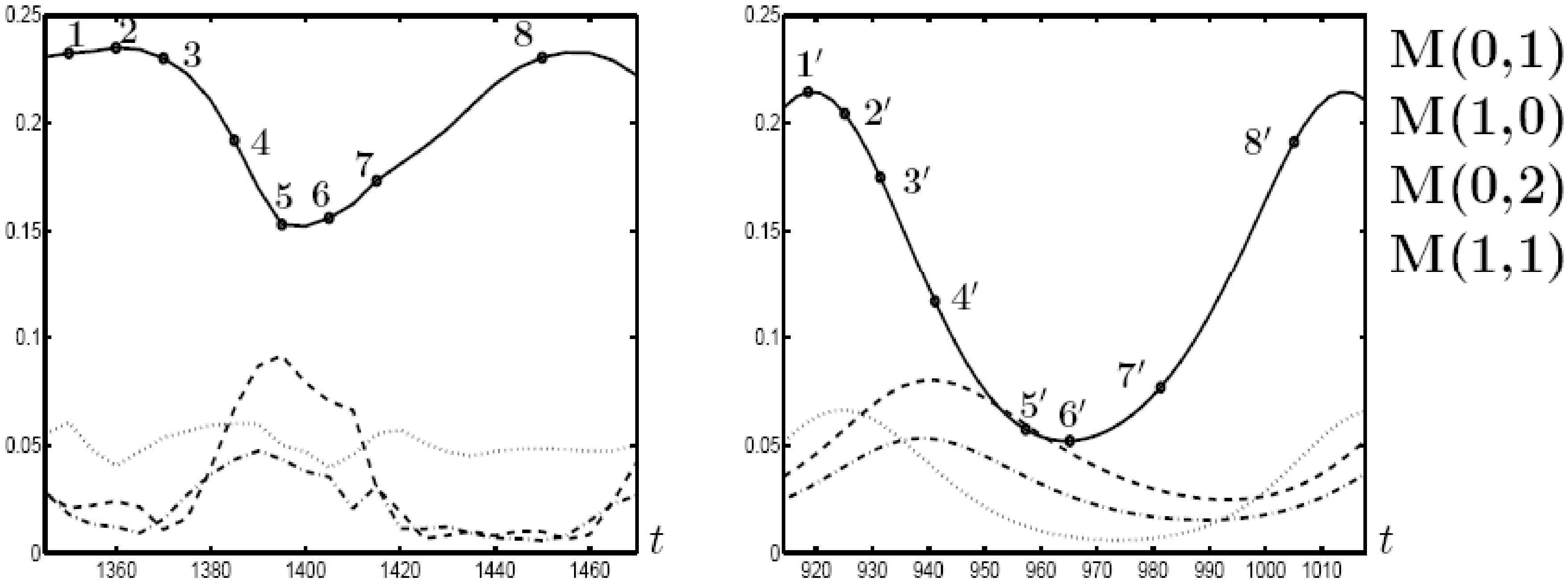
Minimal Flow Unit: $L_x = 1.75\,\pi, L_z = 1.2\,\pi$

- Closeup of one representative period



Minimal Flow Unit: $L_x = 1.75\,\pi, L_z = 1.2\,\pi$

- comparison of DNS and model
  - RMS modal velocities



$T \approx 80 - 100$

$T = 95$