

Distributed Control in Large Actuator/Sensor Arrays

Bassam Bamieh

Department of Mechanical Engineering
University of California, Santa Barbara

<http://www.engineering.ucsb.edu/~bamieh>

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Large Arrayed Systems of Sensors and Actuators

- **New (and old) technologies**

- Micro-Electro-Mechanical-Systems (MEMS) → Large Arrays
- Vehicular Platoons
- Cross Directional (CD) control in pulp and paper processes

- **Modeling and control issues**


- Complexity (Control-Oriented Modeling)
- Overall *System Design* (vs. individual device design)
- **Controller architecture**

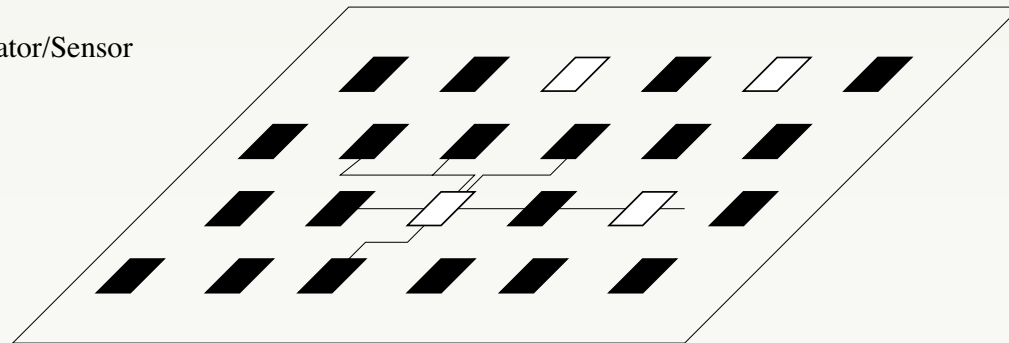
- **Distributed Systems Theory**

- Infinite-dimensional systems with special structure
- **Controller architecture**

Arrays of Micro-Electro-Mechanical-Systems (MEMS)

 : Control Unit

 : Actuator/Sensor



CURRENTLY FEASIBLE: Very large arrays of MEMS with integrated control circuitry

Issues:

- **Tightly coupled dynamics**



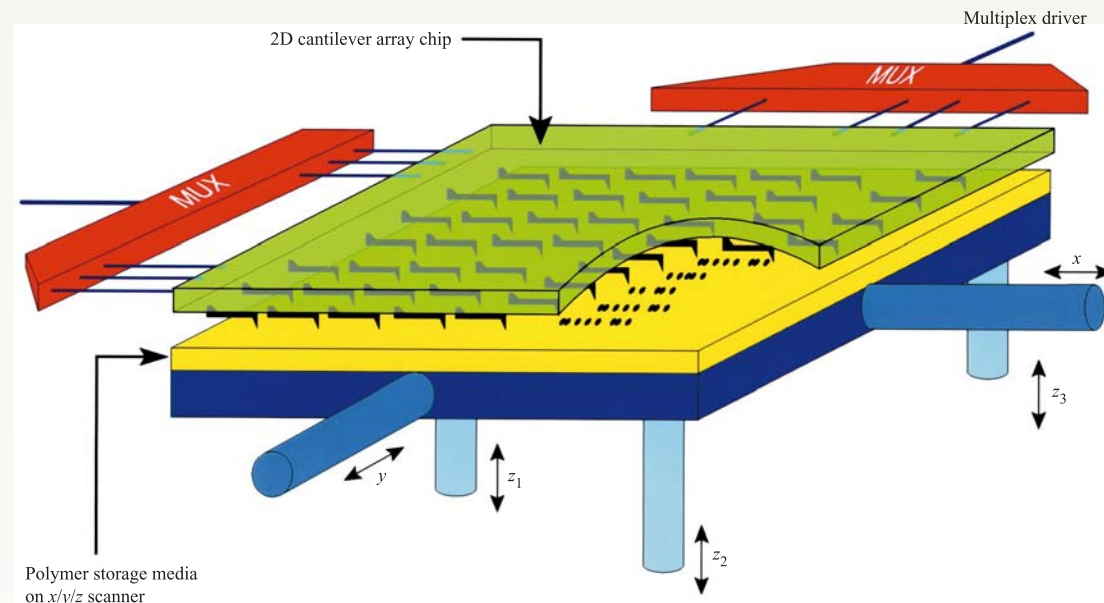
Current designs avoid this with large spacing

Spatio-temporal
instabilities
(e.g. *string instability*)

- **Controller architecture**

- Layout of sensors/actuators
- Communication between actuators/sensors
how to decentralize or localize

Example: Massively Parallel Data Storage (IBM Millpede project)

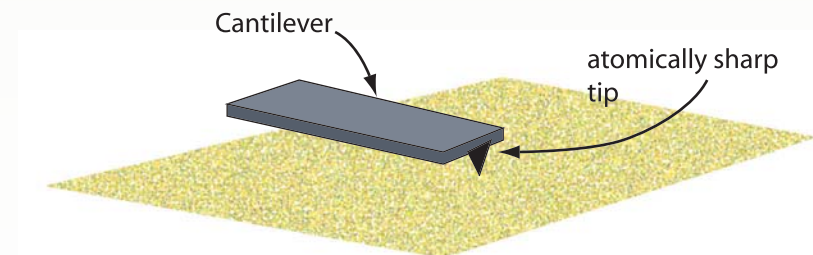


Atomic level resolution using Atomic Force Microscopy (AFM) and Scanning Tunneling

- Microscopy (STM) techniques

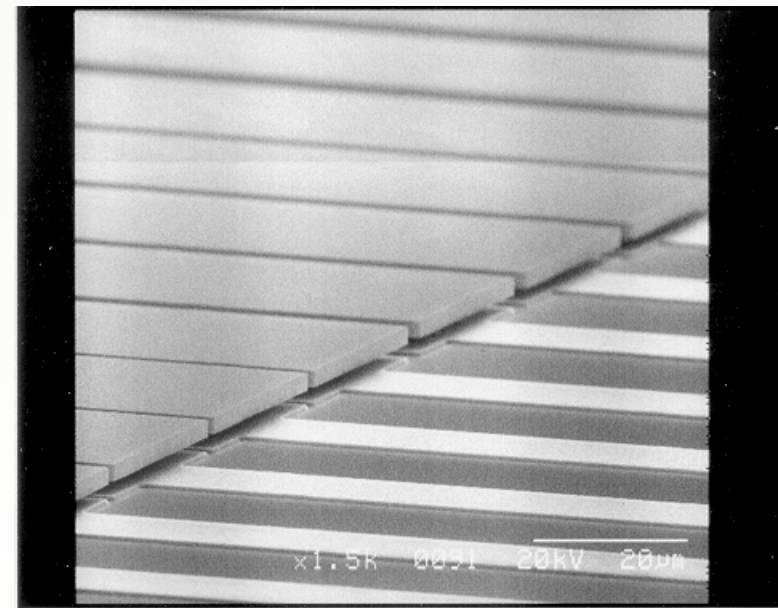
$100 \sim 1000 \text{ Tb/in}^2$ density possible!

- **Problem:** slow scans = low throughput
Solution: go massively parallel



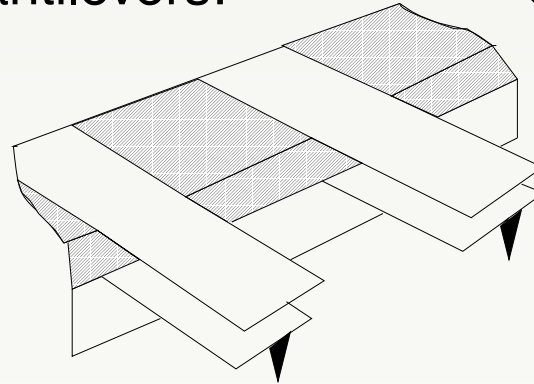
Design and Control Issues in MEMS Arrays

- More tightly packed arrays \longrightarrow more dynamical coupling
 - Micro-cantilever arrays
 - Micro-mirror arrays
- Current fixes:
 - Large spacings
 - Complex design to isolate elements
- Experimental effort at UCSB:
design deliberately coupled arrays
- Demonstrate “electronic” decoupling
using [feedback](#)



Micro-cantilever Array Control

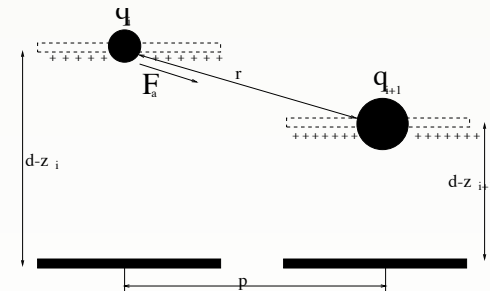
Capacitively actuated micro-cantilevers:



Combined actuator and sensor

Important Considerations:

- Higher throughput, faster “access time” → Tightly packed cantilevers
- For tightly packed cantilevers, significant dynamical coupling due to
 - Mechanical coupling
 - Fringe fields
(Napoli & Bamieh, '01)
- Large arrays $\approx 10,000$ devices
⇒ must use localized control



Distributed Systems with Special Structure

- General Infinite-dimensional Systems Theory

- Well posedness issues (semi-group theory)
- Constructive (convergent) approximation techniques

THEME: *Make infinite-dimensional problems look like finite-dimensional ones*

- Special Structure

- Distributed control and measurement (*now more feasible*)
- Regular (lattice) arrangement of devices

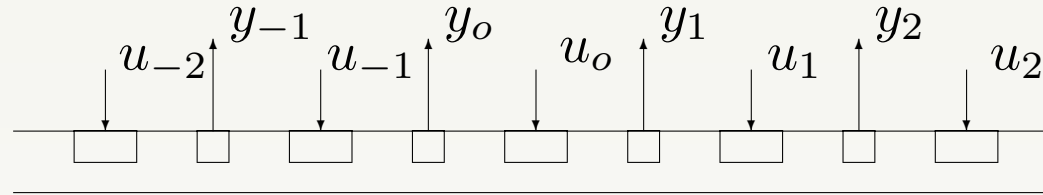
Together \implies *Spatial Invariance*

- Control of “Vehicular Strings”, (Melzer & Kuo, 71)
- Discretized PDEs, (Brockett, Willems, Krishnaprasad, El-Sayed, '74, '81)
- “Systems over rings”, (Kamen, Khargonekar, Sontag, Tannenbaum, ...)
- Systems with “Dynamical Symmetry”, (Fagniani & Willems)

More recently:

- Controller architecture and localization, (Bamieh, Paganini, Dahleh)
- LMI techniques, localization, (D’Andrea, Dullerud, Lall)

Example: Distributed Control of the Heat Equation



u_i : input to heating elements.

y_i : signal from temperature sensor.

Dynamics are given by:

$$\begin{bmatrix} \vdots \\ y_{-1} \\ y_0 \\ y_1 \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots & & & & \vdots \\ \dots & & H_{-1,0} & & \dots \\ & H_{0,-1} & H_{0,0} & H_{0,1} & \\ \dots & & H_{1,0} & & \dots \\ \vdots & & & & \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ u_{-1} \\ u_0 \\ u_1 \\ \vdots \end{bmatrix}$$

Each $H_{i,j}$ is an infinite-dimensional SISO system.

Fact: Dynamics are spatially invariant \Rightarrow H is Toeplitz

The input-output relation can be written as a *convolution over the actuator/sensor index*:

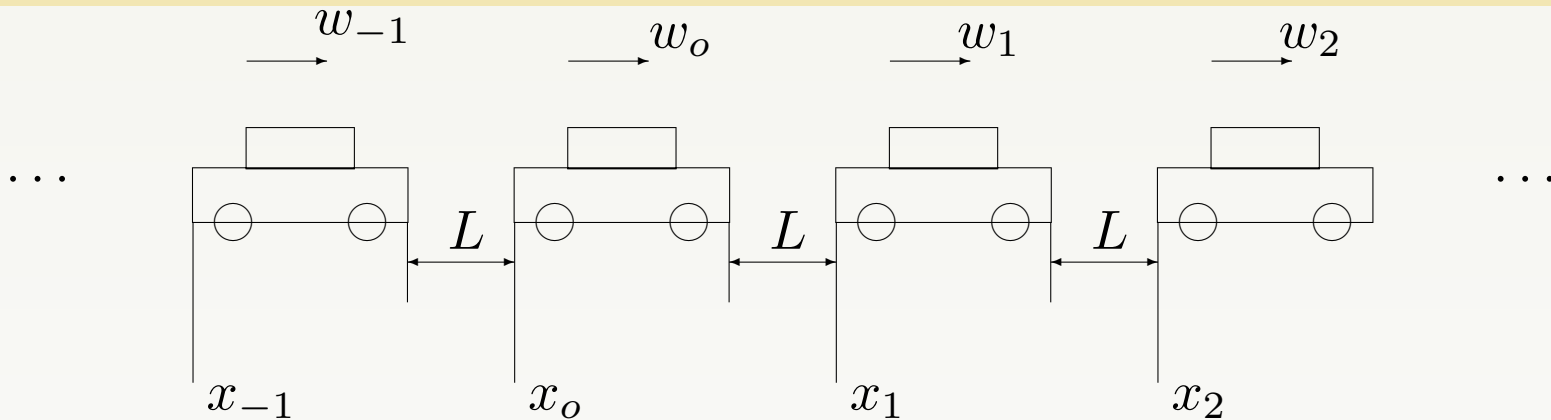
$$y_i = \sum_{j=-\infty}^{\infty} \bar{H}_{(i-j)} u_j,$$

The limit of large actuator sensor array:

$$\frac{\partial \psi}{\partial t}(x, t) = c \frac{\partial^2 \psi}{\partial x^2}(x, t) + u(x, t)$$

$$\psi_x = \int_{-\infty}^{\infty} H_{x-\zeta} u_{\zeta} d\zeta,$$

Vehicular Platoons



Objective: Design a controller for each vehicle to:

- Maintain constant small slot length L .
- Reject the effect of disturbances $\{w_i\}$ (wind gusts, road conditions, etc...)

Warning: Designs based on two vehicle models may lack “string stability”, i.e. disturbances get amplified as they propagate through the platoon.

Problem Structure:

- Actuators: each vehicle’s throttle input.
- Sensors: position and velocity of each vehicle.

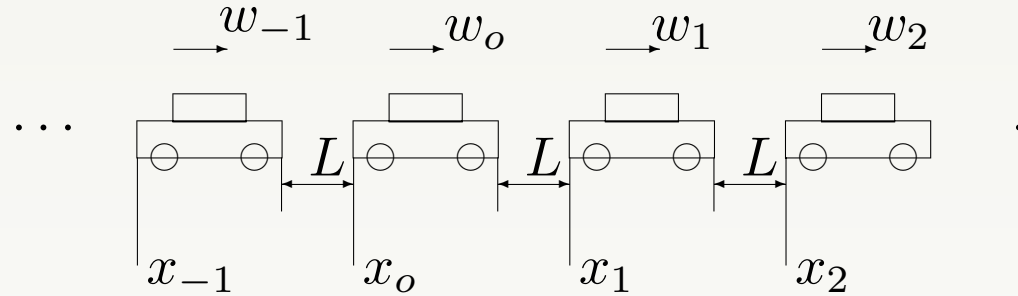
Vehicular Platoons Set-up

x_i : i 'th vehicle's position.

$$\tilde{x}_i := x_i - x_{i-1} - L - C$$

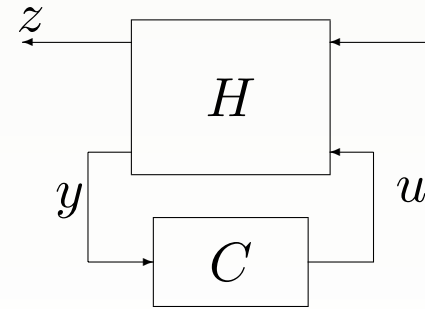
$$\tilde{x}_{1,i} := \tilde{x}_i$$

$$\tilde{x}_{2,i} := \dot{\tilde{x}}_i$$



Structure of generalized plant:

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} \times & \cdots & \times & 0 \\ \times & \cdots & h_o & \cdots \\ 0 & h_1 & \cdots & \cdots \end{bmatrix}$$



$$z = \mathcal{F}(H, C)$$

The generalized plant has a Toeplitz structure!

Optimal Controller for Vehicular Platoon

Example: Centralized \mathcal{H}^2 optimal controller gains for a 50 vehicle platoon
(From: Shu and Bamieh '96)

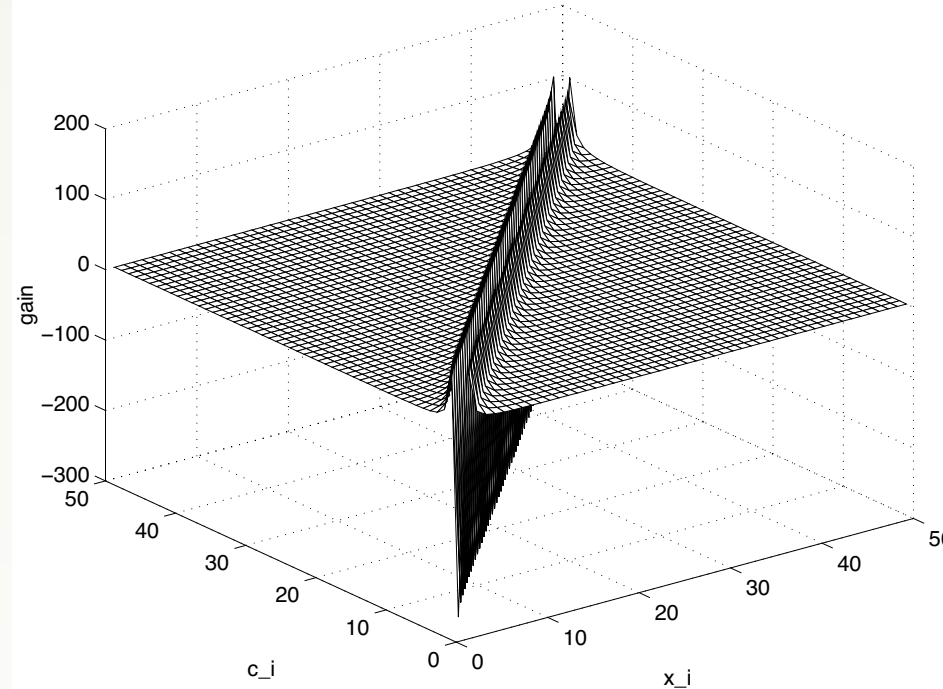


Figure 1: Position error feedback gains for a 50 vehicle platoon

Remarks:

- For large platoons, optimal controller is approximately Toeplitz
- Optimal centralized controller has some inherent decentralization (“localization”)
Controller gains decay away from the diagonal

Q: Do the above 2 results occur in all “such” problems?

Spatial Invariance of Dynamics

Indexing of actuator and sensor signals:

$$u_i(t) := u_{(i_1, \dots, i_n)}(t), \quad y_i(t) := y_{(i_1, \dots, i_n)}(t).$$

$$i := (i_1, \dots, i_n) \text{ a spatial multi-index,} \quad i \in \mathbb{G} := \mathbb{G}_1 \times \dots \times \mathbb{G}_n.$$

Linear input-output relations:

A general linear system from u to y :

$$y_i = \sum_{j \in \mathbb{G}} H_{i,j} u_j, \quad \Leftrightarrow \quad y_{(i_1, \dots, i_n)} = \sum_{j_1 \in \mathbb{G}_1} \dots \sum_{j_n \in \mathbb{G}_n} H_{(i_1, \dots, i_n), (j_1, \dots, j_n)} u_{(j_1, \dots, j_n)},$$

Spatial Invariance:

Assumption 1: Set of spatial indices = commutative group

$$\mathbb{G} := \mathbb{G}_1 \times \dots \times \mathbb{G}_n, \quad \text{each } \mathbb{G}_i \text{ a commutative group.}$$

Remark: “spatial shifting” of signals

$$(S_\sigma u)_i := u_{i-\sigma} \quad \text{Compare with: } \textit{Time shift by } \tau \quad (S_\tau u)(t) := u(t - \tau)$$

Assumption 2: Spatial invariance \longleftrightarrow Commute with spatial shifts

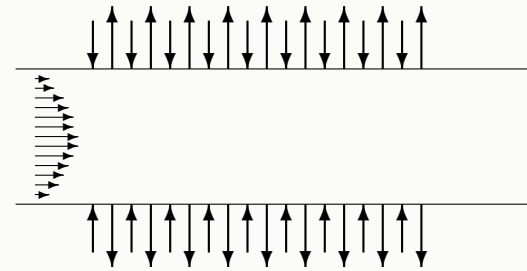
$$\forall \sigma \in \mathbb{G}, \quad H S_\sigma = S_\sigma H \quad \Leftrightarrow \quad S_\sigma^{-1} H S_\sigma = H$$

Examples of Spatial Invariance

Generally: Spatial invariance easily ascertained from basic physical symmetry!

- Vehicular platoons: signals index over \mathbb{Z} .
- Channel flow: Signals indexed over $\{0, 1\} \times \mathbb{Z}$:

$$y_{(l,i)} = \sum_{j=-\infty}^{\infty} H_{(l-0,i-j)} u_{(0,j)} + \sum_{j=-\infty}^{\infty} H_{(l-1,i-j)} u_{(1,j)}, \quad l = 0, 1.$$



Remark: The input-output mapping of a spatially invariant system can be rewritten:

$$y_i = \sum_{j \in \mathbb{G}} \bar{G}_{i-j} u_j, \quad \Leftrightarrow \quad y_{(i_1, \dots, i_n)} = \sum_{j_1 \in \mathbb{G}_1} \dots \sum_{j_n \in \mathbb{G}_n} \bar{G}_{(i_1-j_1, \dots, i_n-j_n)} u_{(j_1, \dots, j_n)}.$$

A spatial convolution

Symmetry in Dynamical Systems and Control Design

- Many-body systems always have some inherent dynamical symmetries: e.g. equations of motion are invariant to certain coordinate transformations
- **Question:** Given an unstable dynamical system with a certain symmetry, is it possible to stabilize it with a controller that has the same symmetry? (i.e. without “breaking the symmetry”)
- **Answer:** Yes! (Fagnani & Willems '93)

Remark: Spatial invariance is a dynamical symmetry

This answer applies to optimal design as well

i.e.

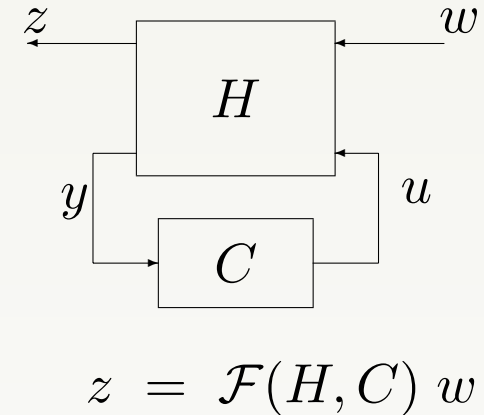
For best achievable performance, need only consider spatially-invariant controllers

The Standard Problem of Optimal and Robust Control

The standard problem:

Signal norms:

$$\|w\|_p^p := \sum_{i \in \mathbb{G}} \int_{\mathbb{R}} |w_i(t)|^p dt = \sum_{i \in \mathbb{G}} \|w\|_p^p$$



Induced system norms:

$$\|\mathcal{F}(G, C)\|_{p \rightarrow i} := \sup_{w \in L^p} \frac{\|z\|_p}{\|w\|_p}.$$

The \mathcal{H}^2 norm:

$$\|\mathcal{F}(G, C)\|_{\mathcal{H}^2}^2 = \|z\|_2^2 = \sum_{i \in \mathbb{G}} \|z_i\|_{L^2}^2,$$

with impulsive disturbance input $w_i(t) = \delta(i)\delta(t)$.

Note: In the platoon problem: finite system norm \Rightarrow string stability.

Spatially-Invariant vs. Spatially-Varying Controllers

Question: Are spatially-varying controllers better than spatially-invariant ones?

Answer: If plant is spatially invariant, no!

LSI := The class of Linear Spatially-Invariant systems.

LSV := The class of Linear Spatially-Varying systems.

Compare the two problems:

$$\gamma_{si} := \inf_{\substack{\text{stabilizing } C \\ C \in LSI}} \|\mathcal{F}(G, C)\|_{p-i}$$

The best achievable performance
with spatially-invariant controllers

$$\gamma_{sv} := \inf_{\substack{\text{stabilizing } C \\ C \in LSV}} \|\mathcal{F}(G, C)\|_{p-i}$$

The best achievable performance
with spatially-varying controllers

Theorem 1. *If the plant and performance objectives are spatially invariant, i.e. if the generalized plant G is spatially invariant, then the best achievable performance can be approached with a spatially invariant controller. More precisely*

$$\gamma_{si} = \gamma_{sv}.$$

Spatially-Invariant vs. Spatially-Varying Controllers (Cont.)

Related Problem: *Time-Varying vs. Time-Invariant Controllers*

Fact: For time-invariant plants, time-varying controllers offer no advantage over time-invariant ones!
for norm minimization problems

Proofs based on use of YJBK parameterization. Convert to

$$\gamma_{ti} := \inf_{\substack{\text{stable } Q \\ Q \in LTI}} \|T_1 - T_2QT_3\| \qquad \gamma_{tv} := \inf_{\substack{\text{stable } Q \\ Q \in LTV}} \|T_1 - T_2QT_3\| ,$$

T_1, T_2, T_3 determined by plant, therefore time invariant.

- The \mathcal{H}^∞ case: (Feintuch & Francis, '85), (Khargonekar, Poolla, & Tannenbaum, '85). *A consequence of Nehari's theorem*
- The ℓ^1 case: (Shamma & Dahleh, '91). *Using an averaging technique*
- Any induced ℓ^p norm: (Chapellat & Dahleh, '92). *Generalization of the averaging technique*

Spatially-Invariant vs. Spatially-Varying Controllers (Cont.)

Idea of proof:

After YJBK parameterization:

$$\gamma_{si} := \inf_{\substack{\text{stable } Q \\ Q \in LSI}} \|T_1 - T_2 Q T_3\| \geq \gamma_{sv} := \inf_{\substack{\text{stable } Q \\ Q \in LSV}} \|T_1 - T_2 Q T_3\|$$

Let \bar{Q} achieve a performance level $\bar{\gamma} = \|T_1 - T_2 \bar{Q} T_3\|$.

Averaging \bar{Q} :

- If \mathbb{G} is finite: define

$$Q_{av} := \frac{1}{|\mathbb{G}|} \sum_{\sigma \in \mathbb{G}} \sigma^{-1} \bar{Q} \sigma. \rightarrow Q_{av} \text{ is spatially invariant, i.e. } \forall \sigma \in \mathbb{G}, \sigma^{-1} Q_{av} \sigma = Q_{av}$$

Then

$$\begin{aligned} \|T_1 - T_2 Q_{av} T_3\| &= \|T_1 - T_2 \left(\frac{1}{|\mathbb{G}|} \sum_{\sigma \in \mathbb{G}} \sigma^{-1} \bar{Q} \sigma \right) T_3\| = \left\| \frac{1}{|\mathbb{G}|} \sum_{\sigma \in \mathbb{G}} \sigma^{-1} (T_1 - T_2 \bar{Q} T_3) \sigma \right\| \\ &\leq \frac{1}{|\mathbb{G}|} \sum_{\sigma \in \mathbb{G}} \left\| \sigma^{-1} (T_1 - T_2 \bar{Q} T_3) \sigma \right\| = \|T_1 - T_2 \bar{Q} T_3\| \end{aligned}$$

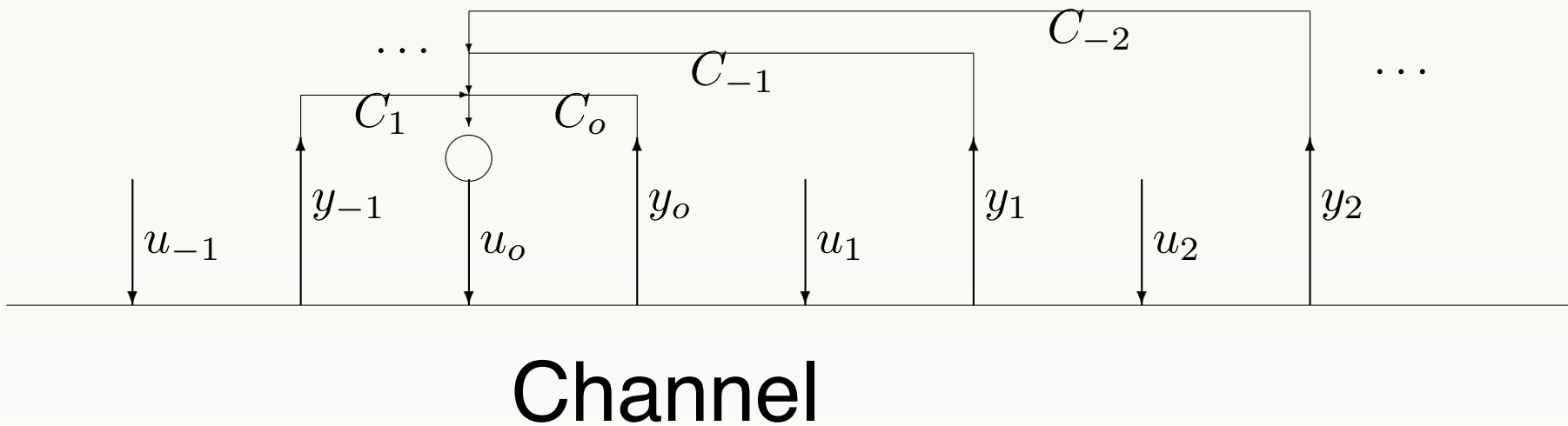
- If \mathbb{G} is infinite, take a sequence of finite subsets $M_1 \subset M_2 \subset \dots$, with $\bigcup_n M_n = \mathbb{G}$,

Then define:
$$Q_n := \frac{1}{|M_n|} \sum_{\sigma \in M_n} \sigma^{-1} \bar{Q} \sigma.$$

Q_n converges weak $*$ to a spatially-invariant Q_{av} with the required norm bound.

Implications of the Structure of Spatial Invariance

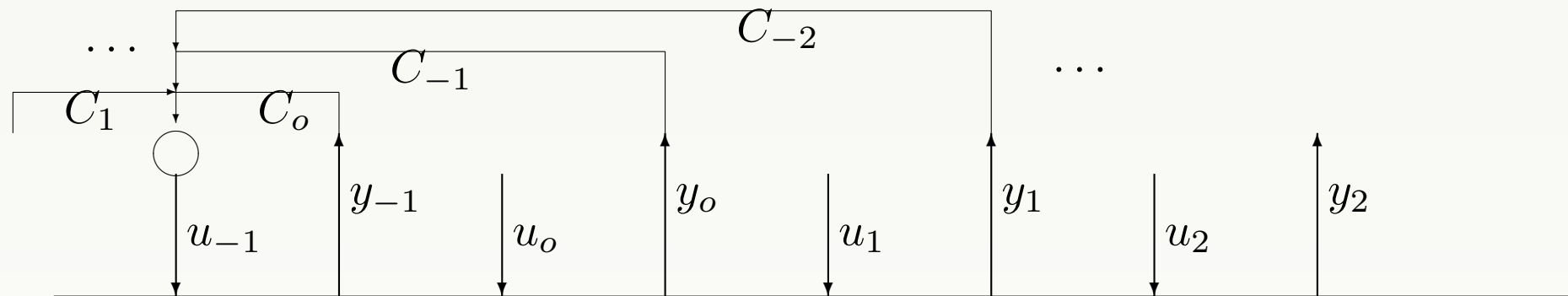
Poiseuille flow stabilization:



$$u_i = \sum_j C_{i-j} y_j$$

Implications of the Structure of Spatial Invariance

Poiseuille flow stabilization:



Channel

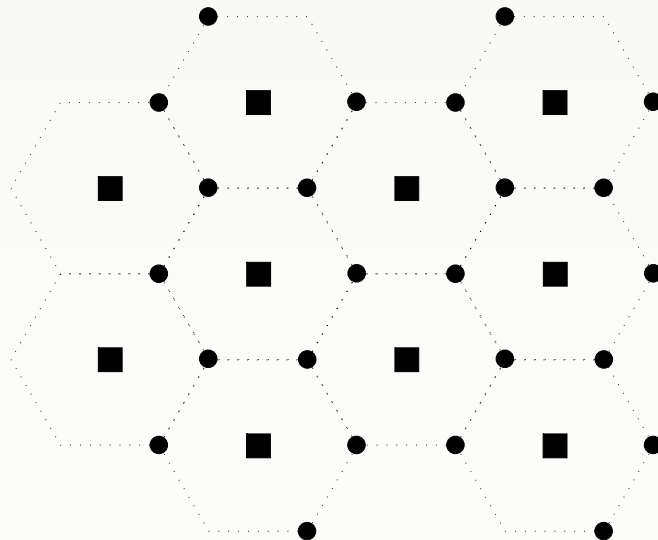
$$u_i = \sum_j C_{i-j} y_j$$

Implications of the Structure of Spatial Invariance (Cont.)

Uneven distribution of sensors and actuators

Consider the following geometry of sensors and actuators:

- Sensor
- Actuator



What kind of spatial invariance do optimal controllers have?

Transform Methods

Consider the following PDE with distributed control:

$$\begin{aligned}\frac{\partial \psi}{\partial t}(x_1, \dots, x_n, t) &= \mathcal{A} \left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n} \right) \psi(x_1, \dots, x_n, t) + \mathcal{B} \left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n} \right) u(x_1, \dots, x_n, t) \\ y(x_1, \dots, x_n, t) &= \mathcal{C} \left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n} \right) \psi(x_1, \dots, x_n, t),\end{aligned}$$

where $\mathcal{A}, \mathcal{B}, \mathcal{C}$ are matrices of polynomials in $\frac{\partial}{\partial x_i}$.

Consider also combined PDE difference equations such as:

$$\begin{aligned}\frac{\partial \psi}{\partial t}(x_1, \dots, x_m, k_1, \dots, k_n, t) &= \mathcal{A} \left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}, z_1^{-1}, \dots, z_n^{-1} \right) \psi(x_1, \dots, x_n, k_1, \dots, k_n, t) \\ &+ \mathcal{B} \left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}, z_1^{-1}, \dots, z_n^{-1} \right) u(x_1, \dots, x_n, k_1, \dots, k_n, t)\end{aligned}$$

We only require that the spatial variables x, k , belong to a commutative group

Taking the Fourier transform:

$$\hat{\psi}(\lambda, t) := \int_{\mathbb{G}} e^{-j\langle \lambda, x \rangle} \psi(x, t) dx,$$

The above system equations become:

$$\frac{d\hat{\psi}}{dt}(\lambda, t) = \mathcal{A}(\lambda) \hat{\psi}(\lambda, t) + \mathcal{B}(\lambda) \hat{u}(\lambda, t)$$

$$\hat{y}(\lambda, t) = \mathcal{C}(\lambda) \hat{\psi}(\lambda, t),$$

where $\lambda \in \hat{\mathbb{G}}$, the dual group to \mathbb{G} .

Remark: This can be thought of as a parameterized family of finite-dimensional systems.

BLOCK DIAGONALIZATION BY FOURIER TRANSFORMS

The Fourier transform converts:

spatially-invariant operators on $\mathcal{L}_2(\mathbb{G})$ \longrightarrow multiplication operators on $\mathcal{L}_2(\hat{\mathbb{G}})$

In general:

group: \mathbb{G}	dual group: $\hat{\mathbb{G}}$	Transform
\mathbb{R}	\mathbb{R}	Fourier Transform
\mathbb{Z}	$\partial\mathbb{D}$	Z-Transform
$\partial\mathbb{D}$	\mathbb{Z}	Fourier Series
\mathbb{Z}_n	\mathbb{Z}_n	Discrete Fourier Transform

and the transforms preserve \mathcal{L}_2 norms:

$$\|f\|_2^2 = \int_{\mathbb{G}} |f(x)|^2 dx = \int_{\hat{\mathbb{G}}} |\hat{f}(\lambda)|^2 d\lambda = \|\hat{f}\|_2^2$$

The system operation is then spatially decoupled or “block diagonalized”:

$$\begin{aligned} \frac{\partial}{\partial t} \psi(x, t) &= A \psi(x, t) + B u(x, t) \\ y(x, t) &= C \psi(x, t) + D u(x, t) \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \hat{\psi}(\lambda, t) &= \hat{A}(\lambda) \hat{\psi}(\lambda, t) + \hat{B}(\lambda) \hat{u}(\lambda, t) \\ \hat{y}(\lambda, t) &= \hat{C}(\lambda) \hat{\psi}(\lambda, t) + \hat{D}(\lambda) \hat{u}(\lambda, t) \end{aligned}$$

\longrightarrow

A distributed,
spatially-invariant system

A parameterized family
of finite-dimensional systems

TRANSFORM METHODS

In physical space

$$\begin{aligned}\frac{d}{dt}\psi_n &= A_n \star \psi_n + B_n \star u_n \\ y_n &= C_n \star \psi_n\end{aligned}$$

After spatial Fourier trans. (FT)

$$\begin{aligned}\frac{d}{dt}\hat{\psi}(\theta) &= \hat{A}(\theta) \hat{\psi}(\theta) + \hat{B}(\theta) \hat{u}(\theta) \\ \hat{y}(\theta) &= \hat{C}(\theta) \hat{\psi}(\theta)\end{aligned}$$

IMPLICATIONS

- Dynamics are decoupled by FT *(The A, B, C operators are “diagonalized”)*
- Quadratic forms preserved by FT \Rightarrow Quadratically optimal control problems are equivalent for FT
- Yields a parametrized family of mutually independent problems

TRANSFER FUNCTIONS

operator-valued transfer function

$$\mathcal{H}(s) = \mathcal{C} (sI - \mathcal{A})^{-1} \mathcal{B}$$

spatio-temporal transfer function

$$H(s, \theta) = \hat{C}(\theta) \left(sI - \hat{A}(\theta) \right)^{-1} \hat{B}(\theta)$$

A multi-dimensional system with temporal, but not spatial causality

Simple Example; Distributed LQR Control of Heat Equation

$$\frac{\partial}{\partial t}\psi(x, t) = c\frac{\partial^2}{\partial x^2}\psi(x, t) + u(x, t) \quad \longrightarrow \quad \frac{d}{dt}\hat{\psi}(\lambda, t) = -c\lambda^2\hat{\psi}(\lambda, t) + \hat{u}(\lambda, t)$$

Solve the LQR problem with $Q = qI$, $R = I$. The corresponding ARE family:

$$-2c\lambda^2 \hat{p}(\lambda) - \hat{p}(\lambda)^2 + q = 0,$$

and the positive solution is:

$$\hat{p}(\lambda) = -c\lambda^2 + \sqrt{c^2\lambda^4 + q}.$$

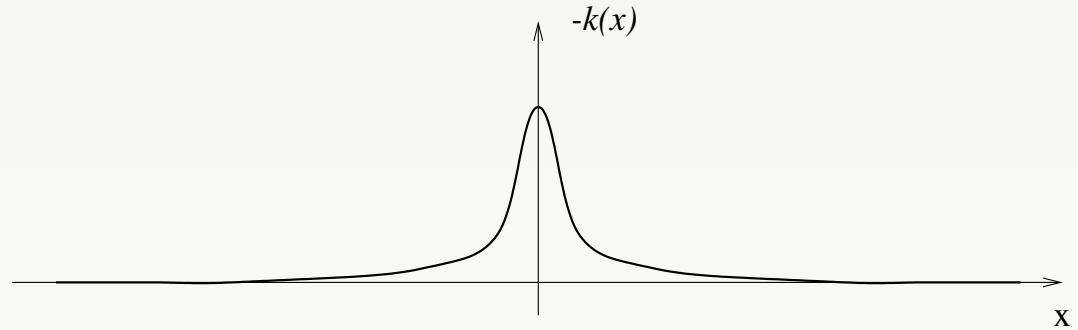
Remark: In general $\hat{P}(\lambda)$ an irrational function of λ , even if $\hat{A}(\lambda)$, $\hat{B}(\lambda)$ are rational.

i.e. PDE systems have optimal feedbacks which are *not* PDE operators.

Let $\{k(x)\}$ be the inverse Fourier transform of the function $\{-\hat{p}(\lambda)\}$.

Then *optimal (temporally static) feedback*

$$u(x, t) = \int_{\mathbb{R}} k(x - \xi) \psi(\xi, t) d\xi$$



Remark: The “spread” of $\{k(x)\}$ indicates information required from distant sensors.

Distributed LQR Control of Heat Equation (Cont.)

Important Observation: $\{k(x)\}$ is “localized”. It decays exponentially!!

$$\hat{k}(\lambda) = c\lambda^2 - \sqrt{c^2\lambda^4 + q}.$$

This can be analytically extended by:

$$\hat{k}_e(s) = cs^2 - \sqrt{c^2s^4 + q},$$

which is analytic in the strip

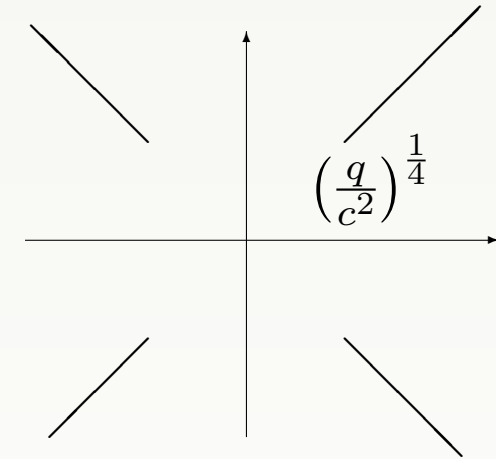
$$\left\{ s \in \mathbb{C} ; \operatorname{Im}\{s\} < \frac{\sqrt{2}}{2} \left(\frac{q}{c^2}\right)^{\frac{1}{4}} \right\}.$$

Therefore: $\exists M$ such that

$$|k(x)| \leq M e^{-\alpha|x|}, \quad \text{for any } \alpha < \frac{\sqrt{2}}{2} \left(\frac{q}{c^2}\right)^{\frac{1}{4}}.$$

This results is true in general: under mild conditions

Solutions of AREs always inverse transform to exponentially decaying convolution kernels



Parameterized ARE solutions yield “localized” operators!

Consider unbounded domains, i.e. $\mathbb{G} = \mathbb{R}$ (or \mathbb{Z}).

Theorem 2. Consider the parameterized family of Riccati equations:

$$A^*(\lambda)P(\lambda) + P(\lambda)A(\lambda) - P(\lambda)B(\lambda)R(\lambda)B^*(\lambda)P(\lambda) + Q(\lambda) = 0, \quad \lambda \in \hat{\mathbb{G}}.$$

Under mild conditions:

there exists an analytic continuation $P(s)$ of $P(\lambda)$ in a region

$$\{|Im(s)| < \alpha\}, \quad \alpha > 0.$$

Convolution kernel resulting from Parameterized ARE has exponential decay.

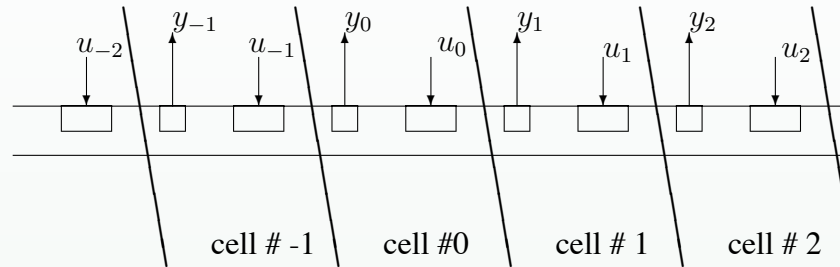
That is, they have some degree of inherent decentralization (“*localization*”)!

Comparison:

- **Modal truncation:** In the transform domain, ARE solutions decay algebraically.
- **Spatial truncation:** In the spatial domain, convolution kernel of ARE solution decays exponentially.

Therefore: Use transform domain to design $\forall \lambda$. Approximate in the spatial domain!

EXAMPLE: one dimensional array of systems indexed in \mathbb{Z} .



ψ_n : the state of the system in the n 'th cell

total state: $\{\dots, \psi_{-1}, \psi_0, \psi_1, \dots\}$

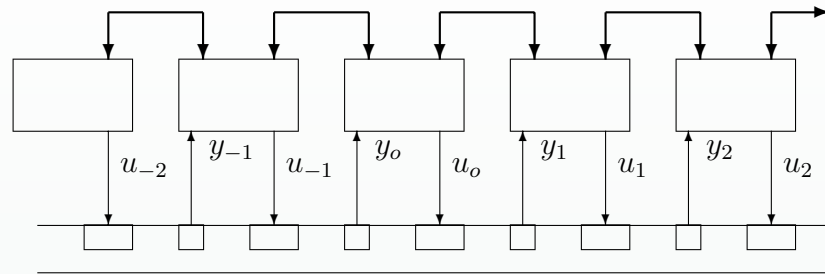
$$\frac{d}{dt}\psi_n = \sum_m A_{n-m} \psi_m + \sum_m B_{n-m} u_m$$

$$y_n = \sum_m C_{n-m} \psi_m$$



$$\frac{d}{dt}\psi_n = A_n \star \psi_n + B_n \star u_n$$

$$y_n = C_n \star \psi_n$$



Observer based controller has the following structure:

Plant

$$\begin{aligned} \frac{d}{dt}\psi_n &= A_n \star \psi_n + B_n \star u_n \\ y_n &= C_n \star \psi_n \end{aligned}$$

Controller

$$\begin{aligned} u_i &= K_i \star \hat{\psi}_i \\ \frac{d}{dt}\hat{\psi}_n &= A_n \star \hat{\psi}_n + B_n \star u_n \\ &\quad + L_n \star (y_n - \hat{y}_n) \end{aligned}$$

REMARKS:

- Optimal Controller is “locally” finite dimensional.
- The gains $\{K_i\}$, $\{L_i\}$ are localized (exponentially decaying) \rightarrow “spatial truncation”
- After truncation, local controller need only receive information from neighboring subsystems.
- Quadratically optimal controllers are inherently distributed and semi-decentralized (*localized*)

The many remaining issues

- Various heterogeneities
 - Spatial variance
 - Irregular arrangements of sensors and actuators
- How to specify “localization” apriori
- The complexities of “high order”
 - *The phenomenology of linear infinite dimensional systems can be arbitrarily complex*