AC Electrokinetics

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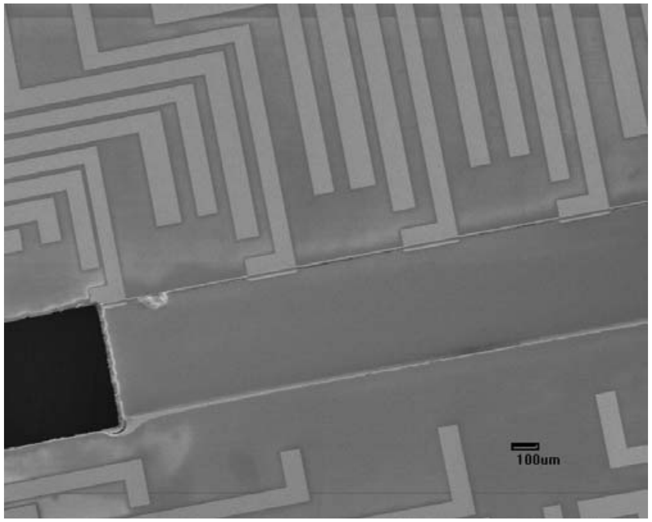
Vortex Structures in Microfluidics

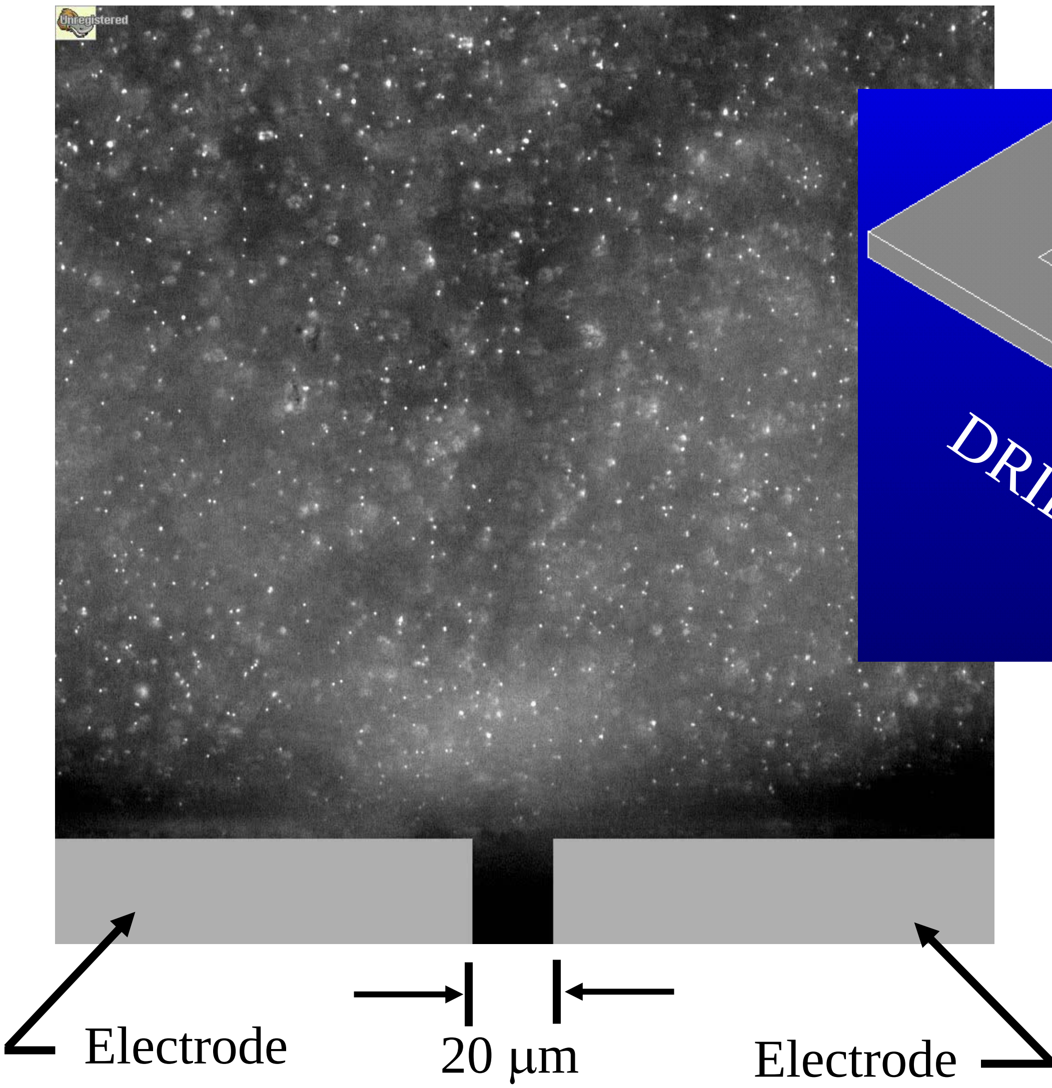
- How are vortex structures possible?
  - Vortex shedding
    - Zero Reynolds number
  - Moving parts
    - Difficult to fabricate
  - DC Electroosmosis
    - Requires large voltages (~.1 - 1 kV)
    - Electrolysis
  - Magnetic fields
    - Restrict to aqueous buffer solutions

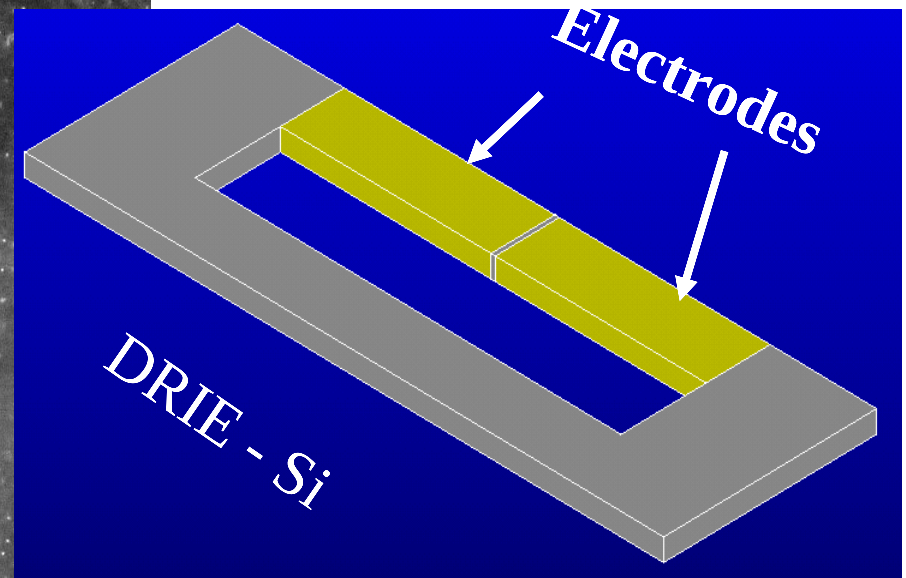
AC Electrokinetics (advantages)

- High frequency electric fields
  - Reduce or eliminate electrolysis
- Place electrodes locally inside microchannel
  - Increased flexibility
  - Locally addressable electrodes
  - **Decrease electric potential**



AC Electric Potential: 6Vrms, 200 kHz

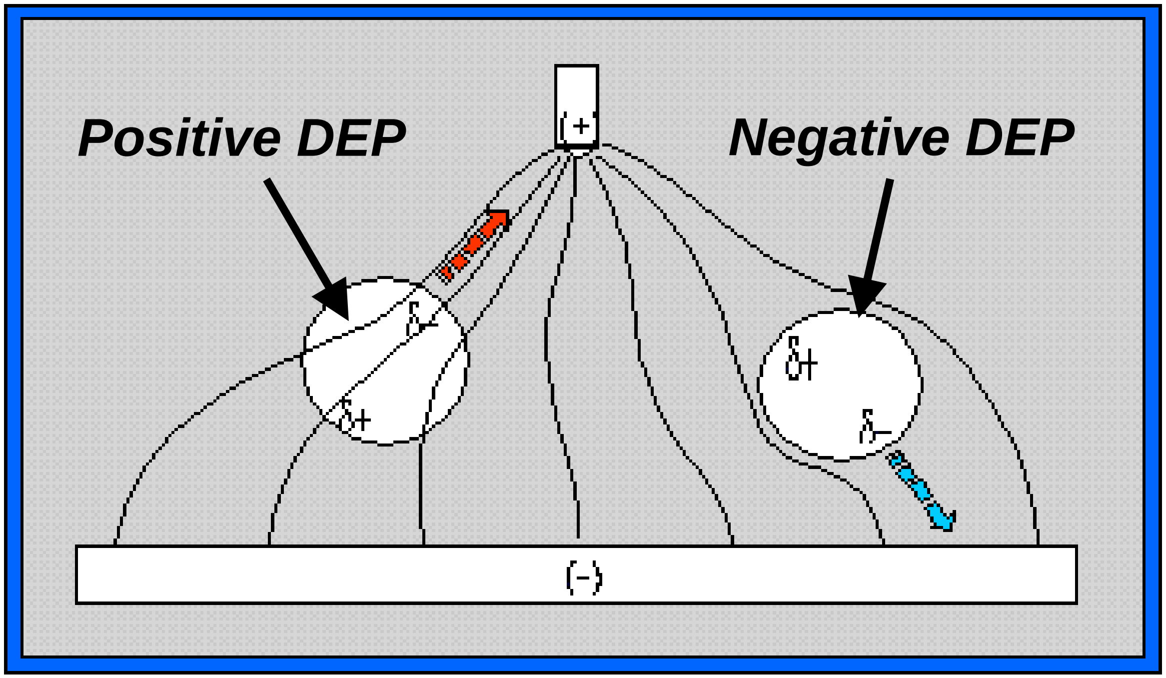




Are we observing:

1. Particle motion or
2. Fluid motion

Particle Motion: Dielectrophoresis (DEP)



http://www.ibmm.informatics.bangor.ac.uk

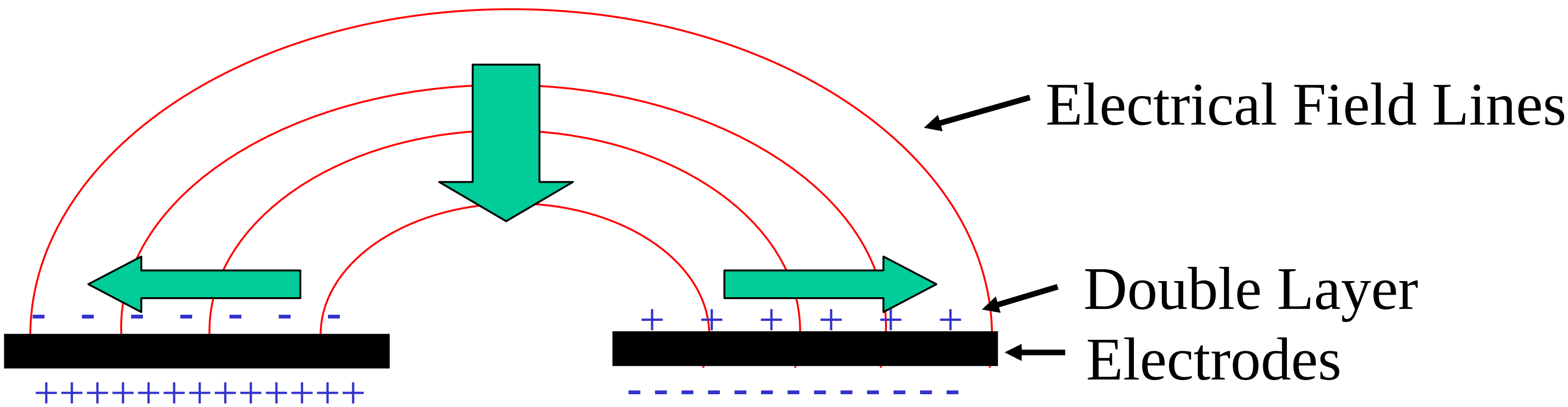
DEP Force

$$\vec{F}_{DEP} = 2\pi\varepsilon_m r^3 \Re\{K\} \nabla|E|^2$$

Stokes Drag

$$\vec{F}_D = 6\pi\mu r\left(\vec{u} - \vec{u}_p\right)$$

Fluid Motion: AC Electroosmosis (Ramos, 1998)



$$u_s = \frac{1}{8}\frac{\varepsilon V_0^2 \Omega^2}{\mu\, r\left(1+\Omega^2\right)^2}$$

Velocity on Electrode Surface, u_s

$$\Omega = \omega\, r \frac{\varepsilon}{\sigma}\frac{\pi}{2}\kappa$$

Non-dimensional frequency

κ inverse of Debye length

Fluid Motion: Electrothermal Effect (Ramos et al., 1998)

Gauss's Law

$$\nabla^2 \phi = \frac{-\rho_q}{\varepsilon_0} \ , \ \vec{E} = -\nabla \phi$$

Ohm's Law

$$\vec{J} = \sigma \vec{E}$$

Electrical Power Dissipation

$$W = \vec{J} \cdot \vec{E}$$

Temperature Field

$$\underbrace{\rho c_p \frac{DT}{Dt}}_{\to 0} = k\nabla^2 T + \underbrace{\sigma E^2}_{\text{Joule Heating}}$$

Fluid Motion: Electrothermal Effect (Ramos et al., 1998)

Body Force on Fluid

$$\vec{f}_E = \underbrace{\rho_q \vec{E}}_{\text{Coulomb Force}} \underbrace{- \frac{1}{2} \left| \vec{E} \right|^2 \nabla \varepsilon}_{\text{Dielectric Force}}$$

Decompose Electric Field

$$\vec{E} = \underbrace{\vec{E}_0}_{\text{Applied Field}} + \underbrace{\vec{E}_1}_{\text{Perturbed Field (charge variation)}}$$

where

$$\left| \vec{E}_1 \right| \ll \left| \vec{E}_0 \right|$$

Fluid Motion: Electrothermal Effect (Ramos et al., 1998)

Body Force on Fluid

$$\vec{f}_E = \rho_q \vec{E} - \frac{1}{2}\left|\vec{E}\right|^2 \nabla \varepsilon$$

(from before)

Gauss's Law

$$\nabla \cdot \left(\varepsilon \vec{E}\right) = \rho_q$$

Expanding

$$\rho_q = \nabla \varepsilon \cdot \vec{E}_0 + \varepsilon \nabla \cdot \vec{E}_1 + hot$$

$$\Rightarrow \quad \boxed{\vec{f}_E = \left(\nabla \varepsilon \cdot \vec{E}_0 + \varepsilon \nabla \cdot \vec{E}_1\right)\vec{E}_0 - \frac{1}{2}\left|\vec{E}_0\right|^2 \nabla \varepsilon}$$

Fluid Motion: Electrothermal Effect (Ramos et al., 1998)

Charge Conservation

$$\frac{\partial \rho_q}{\partial t} = \nabla \cdot \left(\sigma \vec{E} + D\nabla \rho_q + \rho_q \vec{u} \right)$$

(the terms $D\nabla \rho_q$ and $\rho_q \vec{u}$ are struck through, each marked 0)

⇒

$$\frac{\partial \rho_q}{\partial t} = \nabla \cdot \sigma \vec{E}$$

$$\rho_q = \nabla \cdot \left(\varepsilon \vec{E} \right)$$

$$\frac{\partial}{\partial t}\left(\nabla \cdot \varepsilon \vec{E} \right) + \nabla \cdot \left(\sigma \vec{E} \right) = 0$$

Fluid Motion: Electrothermal Effect (Ramos et al., 1998)

$$\frac{\partial}{\partial t}\left(\nabla \cdot \varepsilon \vec{E}\right) + \nabla \cdot \left(\sigma \vec{E}\right) = 0$$

Sinusoidal Fields

$$\vec{E}_0(t) = \mathrm{Re}\left[\vec{E}_0\, e^{j\omega t}\right]$$

$$\frac{\partial}{\partial t}(\quad) = j\omega(\quad)$$

Combining

$$\nabla \cdot \vec{E}_1 = \frac{-\left(\nabla \sigma + j\omega \nabla \varepsilon\right) \cdot \vec{E}_0}{\sigma + j\omega\varepsilon}$$

Fluid Motion: Electrothermal Effect (Ramos et al., 1998)

Force on Fluid

$$\vec{f}_E = \left(\nabla\varepsilon\cdot\vec{E}_0 + \varepsilon\nabla\cdot\vec{E}_1\right)\vec{E}_0 - \frac{1}{2}\left|\vec{E}_0\right|^2\nabla\varepsilon$$

$$\nabla\cdot\vec{E}_1 = \frac{-\left(\nabla\sigma + j\omega\nabla\varepsilon\right)\cdot\vec{E}_0}{\sigma + j\omega\varepsilon}$$

$$\left\langle \vec{f}_E \right\rangle = -0.5\left[\underbrace{\left(\frac{\nabla\sigma}{\sigma} - \frac{\nabla\varepsilon}{\varepsilon}\right)\cdot\overline{E}_0\frac{\varepsilon\overline{E}_0}{1+(\omega\tau)^2}}_{\text{Coulomb}} + \underbrace{0.5\,|E_0|^2\,\nabla\varepsilon}_{\text{Dielectric}}\right]$$

Electrical Properties

$$\frac{\nabla\sigma}{\sigma} = \underbrace{\frac{1}{\sigma}\frac{\partial\sigma}{\partial T}}_{2\%}\nabla T \qquad \frac{\nabla\varepsilon}{\varepsilon} = \underbrace{\frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial T}}_{-0.4\%}\nabla T$$

Electrothermal Effect: Computational Approach

Applied Electric Field

$$\nabla^2 \phi_0 = 0 \ , \ \ \vec{E}_0 = -\nabla \phi_0$$

Temperature Field

$$k \nabla^2 T + \sigma \left| \vec{E}_0 \right|^2 = 0$$

Electrical Properties

$$\frac{\nabla \sigma}{\sigma} = \underbrace{\frac{1}{\sigma} \frac{\partial \sigma}{\partial T}}_{2\%} \nabla T \qquad \frac{\nabla \varepsilon}{\varepsilon} = \underbrace{\frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial T}}_{-0.4\%} \nabla T$$

Electrothermal Effect: Computational Approach

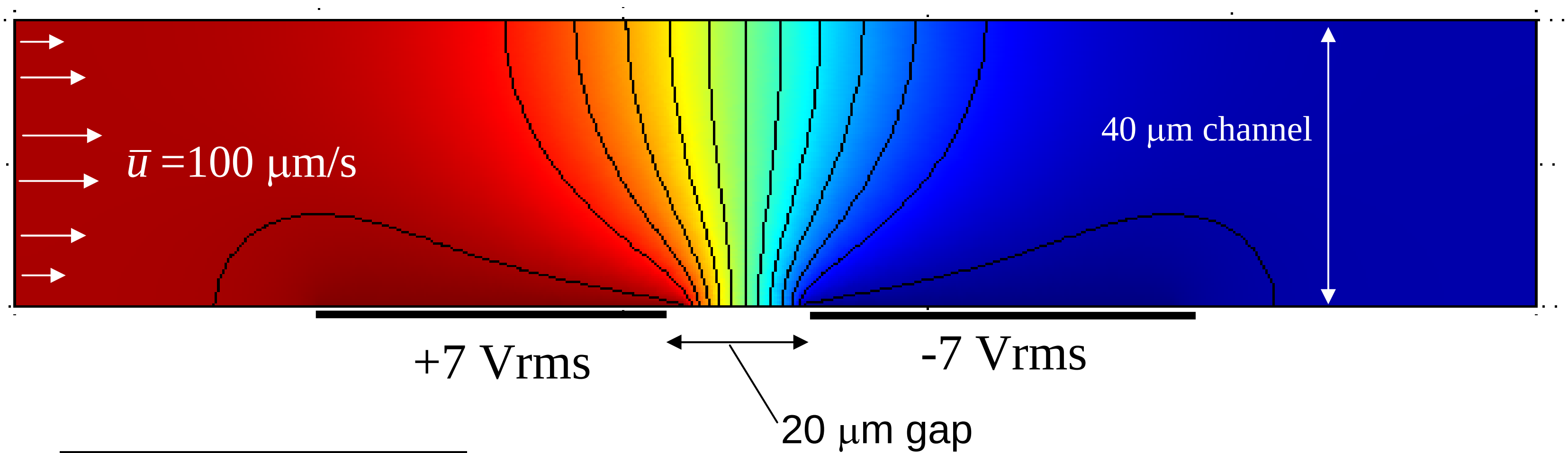
Electrothermal Body Force

$$\left\langle \vec{f}_E \right\rangle = -0.5\left[\underbrace{\left(\frac{\nabla\sigma}{\sigma} - \frac{\nabla\varepsilon}{\varepsilon} \right) \cdot \overline{E}_0 \frac{\varepsilon \overline{E}_0}{1+(\omega\tau)^2}}_{\textbf{Coulomb}} + \underbrace{0.5\,|E_0|^2\,\nabla\varepsilon}_{\textbf{Dielectric}} \right]$$

Stokes' Equation

$$\begin{cases} 0 = -\nabla p + \mu\nabla^2\vec{u} + \left\langle \vec{f}_E \right\rangle \\ \nabla\cdot\vec{u} = 0 \end{cases}$$

Electrothermal Model: Potential field

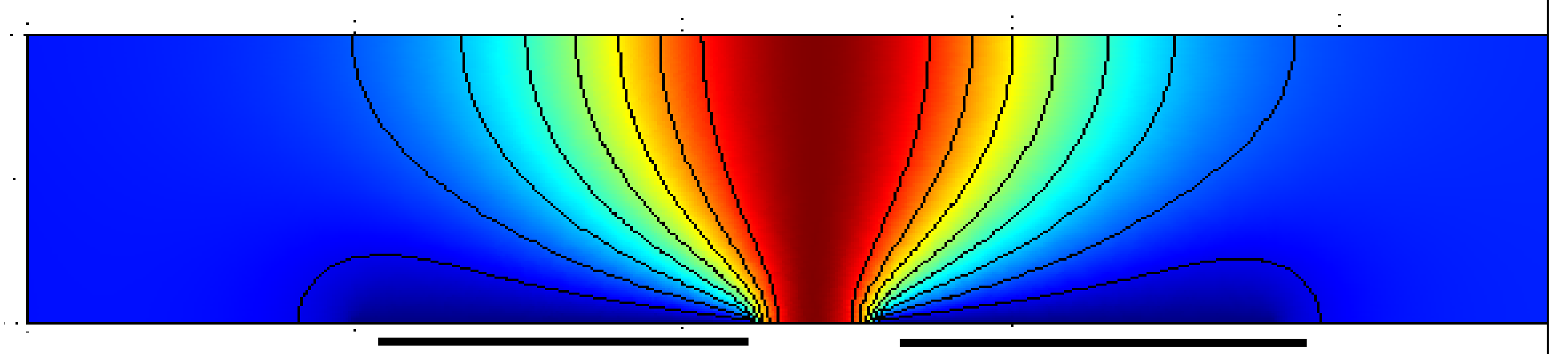


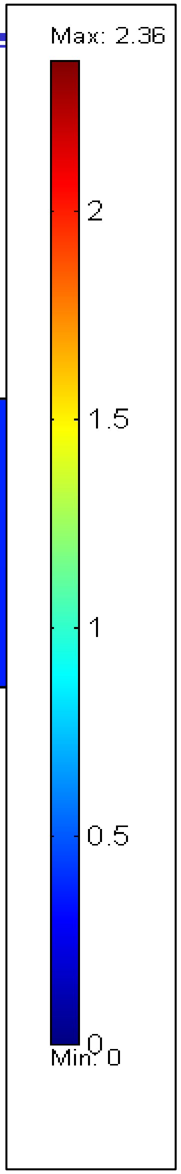
$$\nabla^2 V = 0$$

$$\vec{E} = -\nabla V$$

Electrothermal Model: Temperature

$T - T_{\infty}$





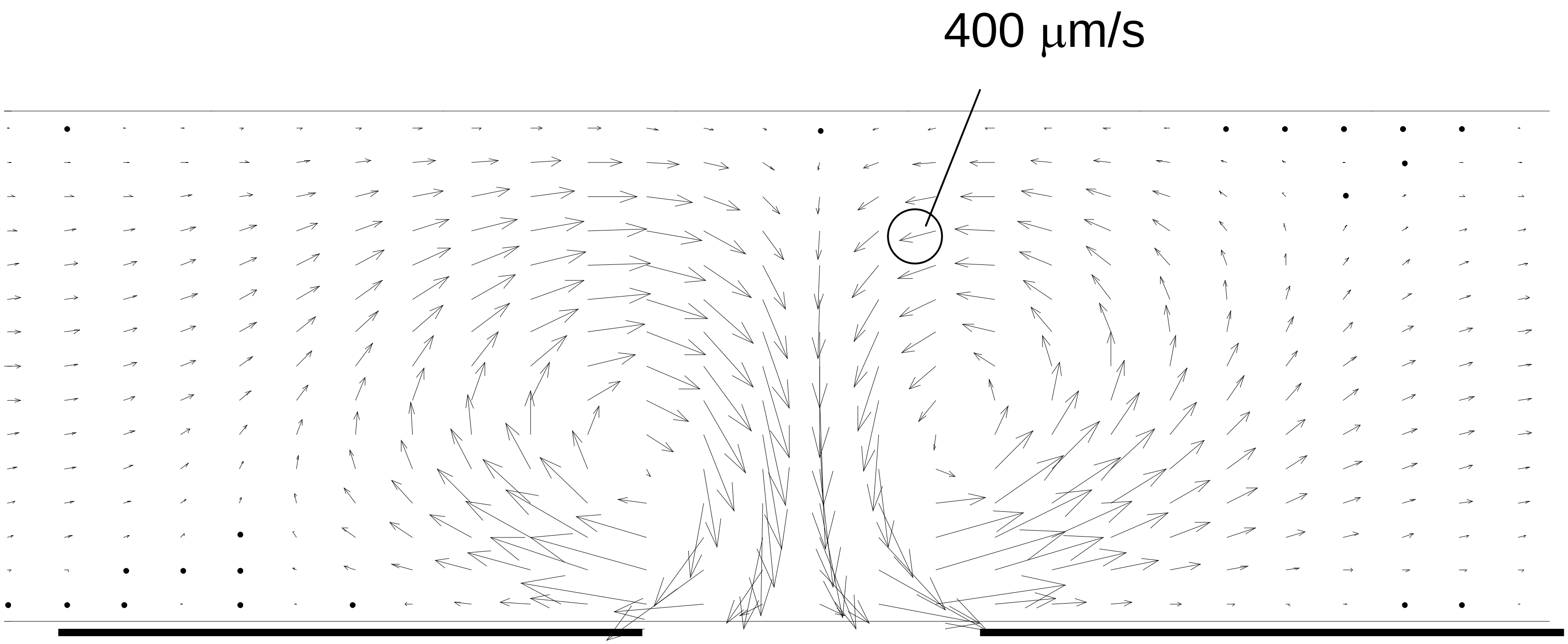
$$\rho c_p \frac{DT}{Dt} = k\nabla^2 T + \sigma E^2$$

0

Heat source

Electrothermal Model : Velocity

Electrothermally modified channel flow (center section only)



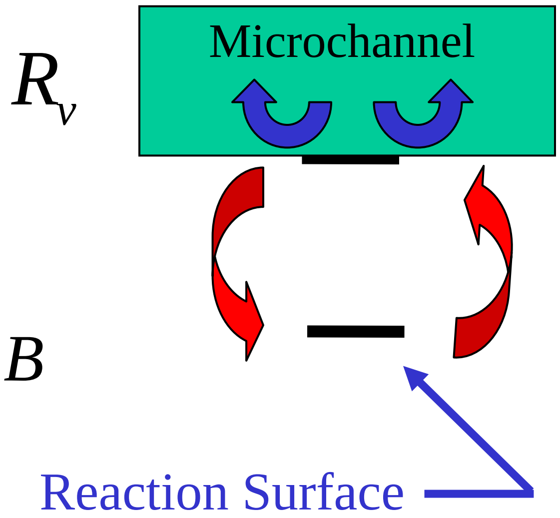
$$0 = -\nabla p + \mu \nabla^2 \vec{u} + \vec{F}_t$$

$$\nabla \cdot \vec{u} = 0$$

Scalar Transport and Reaction Kinetics

Bulk Concentration

$$\frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c = D \nabla^2 c + R_v$$



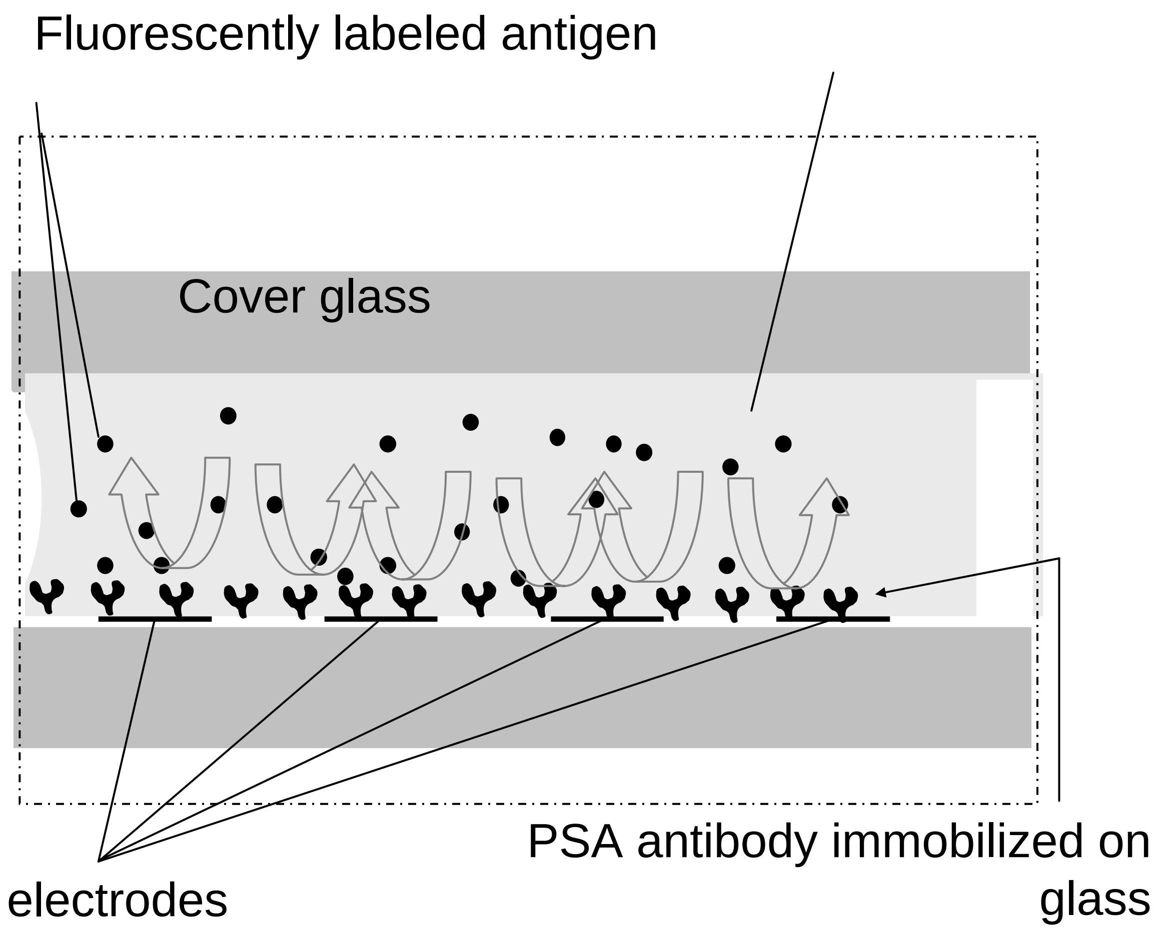
1st Order Heterogeneous Reaction

$$\frac{\partial B}{\partial t} = k_{on}\, c\left(R_T - B\right) - k_{off}\, B$$

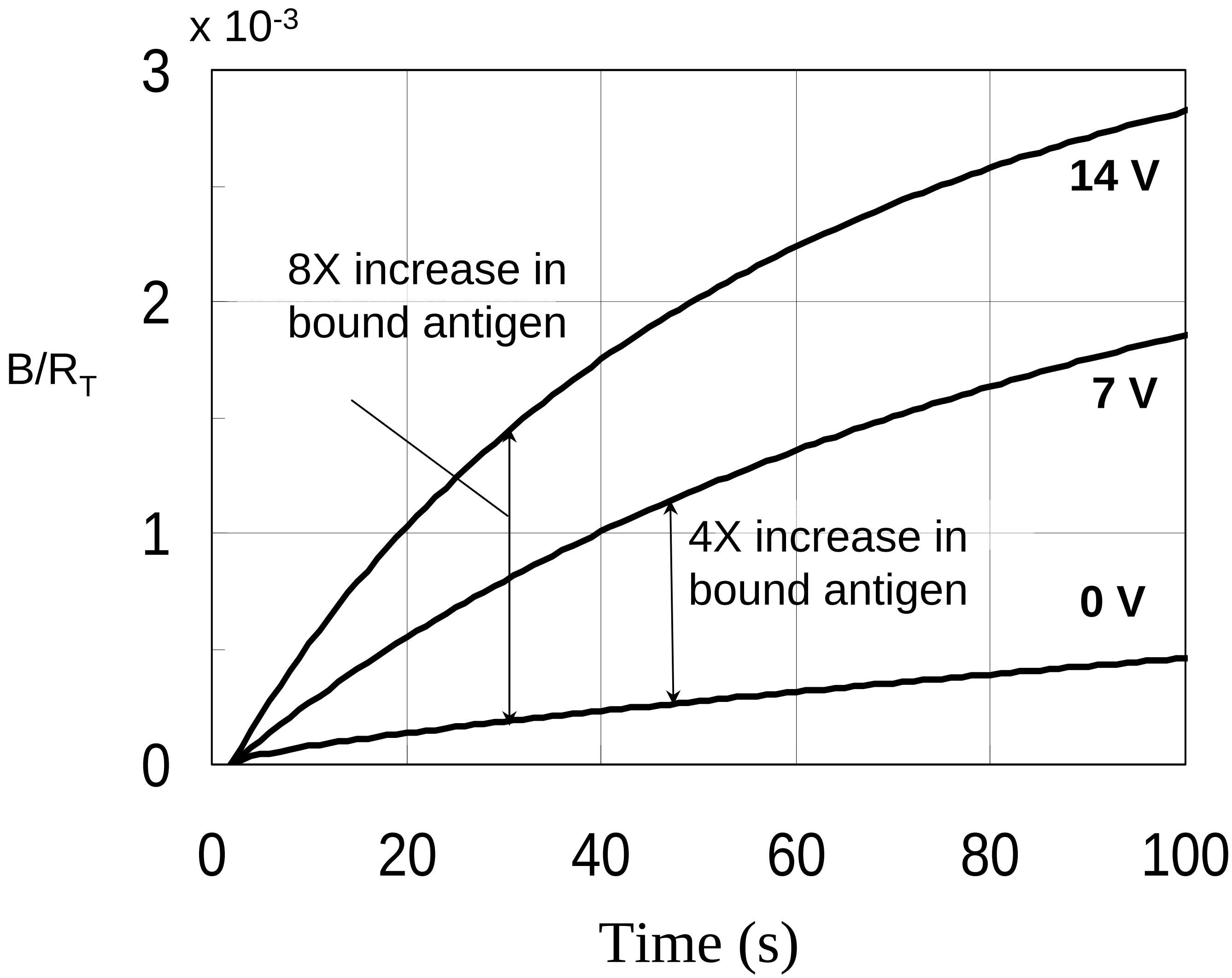
Matching Condition

$$\underbrace{\frac{\partial B}{\partial t}}_{\text{Reaction Rate}} = \underbrace{-D\frac{\partial c}{\partial n}}_{\text{Diffusive Flux}}$$

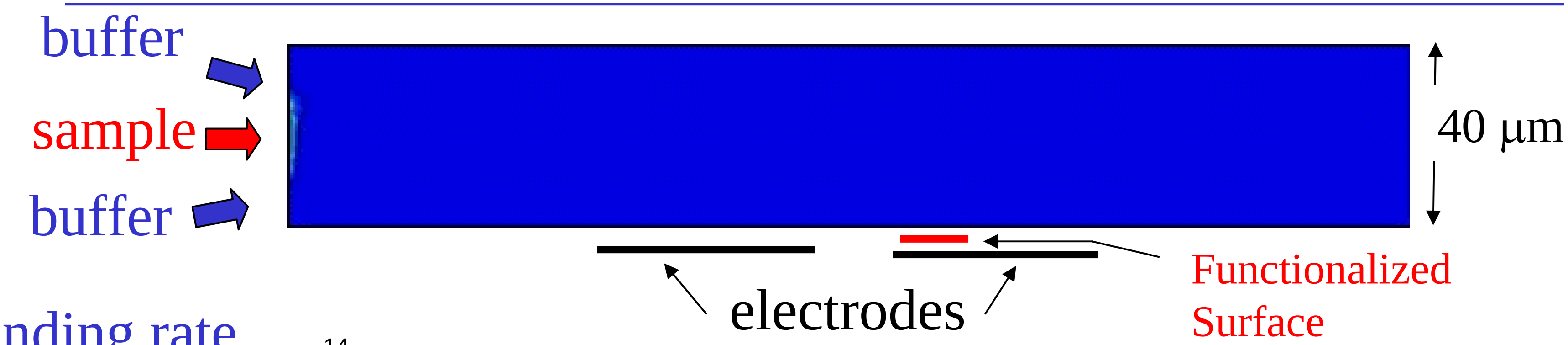
Immunoassay Reactions (ELISA)

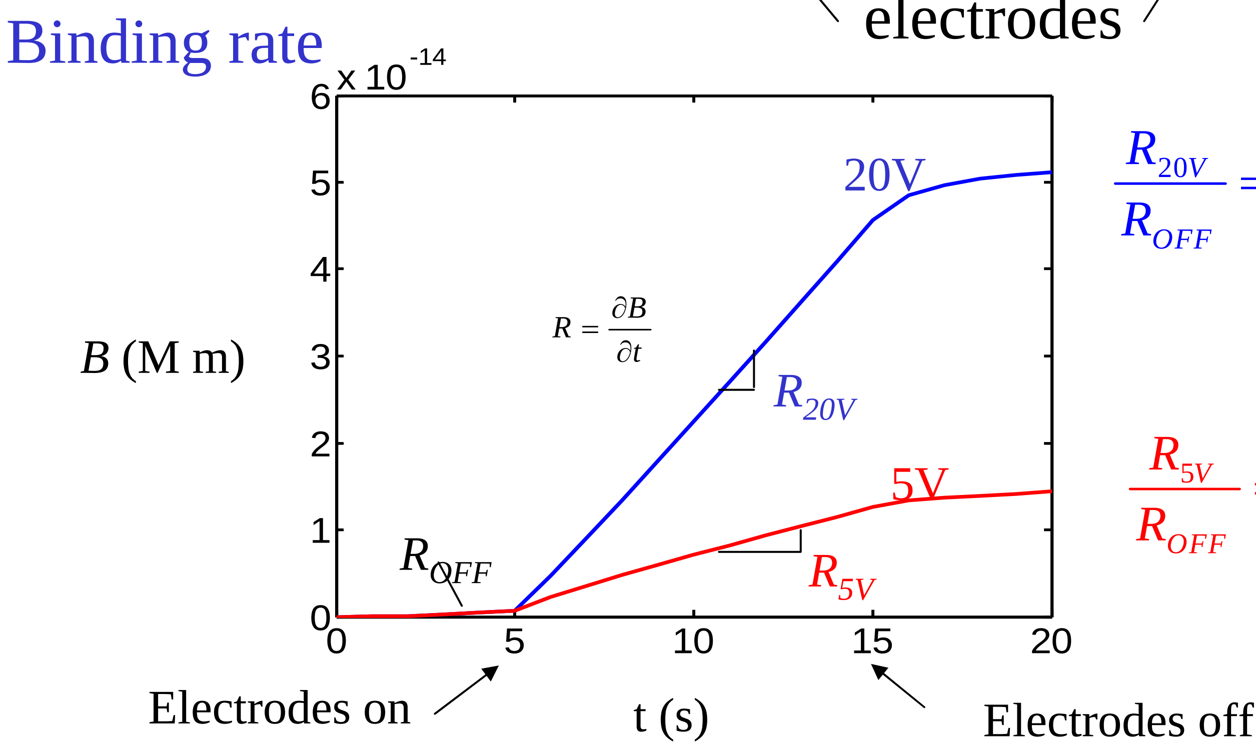


Normalized Bound Concentration (ELISA)



Electrokinetic Switching





$$\frac{R_{20V}}{R_{OFF}} = 21$$

$$\frac{R_{5V}}{R_{OFF}} = 6$$

Multiple Physics

1. Gauss's Law
2. Energy Eq.
3. Conductivity Gradient
4. Navier-Stokes Eq.
5. Convection-Diffusion
6. Heterogeneous React.

See Femlab Example