Electrostatics

11. Spring 04 Electrokinetics

Maxwell's Eqn.

$$\nabla \times \underset{\sim}{H} = \underset{\sim}{J_c} + \cancel{\underset{\sim}{J_D}} \quad \left(\frac{\partial \underset{\sim}{D}}{\partial t}\right)$$

Amperes' Law

$\nearrow J_c$ [induce H]

Displacement current

note: $\underset{\sim}{J_D} = \frac{\partial \underset{\sim}{D}}{\partial t}$

$\underset{\sim}{H}$ ~~magnetic~~ magnetic field strength

$$\nabla \times \underset{\sim}{E} = -\frac{\partial B}{\partial t}$$

Faraday's Law

$\underset{\sim}{B}$ mag. flux density

$$\nabla \cdot \underset{\sim}{D} = \rho_f$$

Gauss' Law

ρ_f : free charge

$\underset{\sim}{D}$: displacement field

Integral form $\oint_{S'} \underset{\sim}{D} \cdot d\underset{\sim}{S'} = \int_V \rho_f \, dV$

$$\nabla \cdot \underset{\sim}{B} = 0$$

non existence of monopole

$$\oint_{S'} \underset{\sim}{B} \cdot dS' = 0$$

$\underset{\sim}{E}$: electric field

$\underset{\sim}{D}$: displacement field [depends on charge, ind. of material]

$\underset{\sim}{H}$: magnetic field strength [depends on charge movement, ind. of material property]

$\underset{\sim}{B}$: mag flux density

for linear isotropic materials:
~~constitutive~~ constitutive relation

$$\boxed{\begin{array}{c} \underset{\sim}{D} = \varepsilon \underset{\sim}{E} \\ \hline \underset{\sim}{B} = \mu \underset{\sim}{H} \end{array}}$$

usually $\mu = \mu_0$ permeability of free space, except ferrous materials

$$\boxed{\varepsilon = \varepsilon_r \varepsilon_0}$$

ε_r relative permittivity $=$ 1 air, 78.3 water

Boundary Conditions

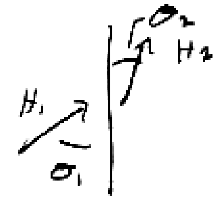
Magnetic Fields

$B_{n1} = B_{n2}$

$H_{t1} = H_{t2}$ [current free]

$(\underset{\sim}{H}_1 - \underset{\sim}{H}_2) \times a_{12} = \underset{\sim}{K}$ [current sheet]

$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r2}}{\mu_{r1}}$



Electric Fields

$D_{n1} = D_{n2}$ (charge-free)

$(\underset{\sim}{D}_1 - \underset{\sim}{D}_2) \cdot a_{n12} = -\rho_s$ (surf. charge)

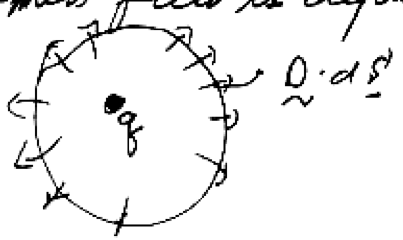
$E_{t1} = E_{t2}$

$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$ (charge-Free)

~~Electric~~

Gauss Law:

Displacement field is defined in terms charge within the volume



$$\boxed{\oint_S \underset{\sim}{D} \cdot d\underset{\sim}{S} = \int_V \rho_s \, dV}$$ Gauss' Law

Electric Field

Defined as the force applied to a test charge, Q_t

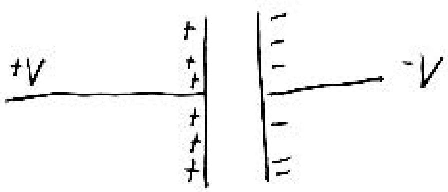
$\underset{\sim}{E} \equiv \underset{\sim}{F_t} / Q_t$

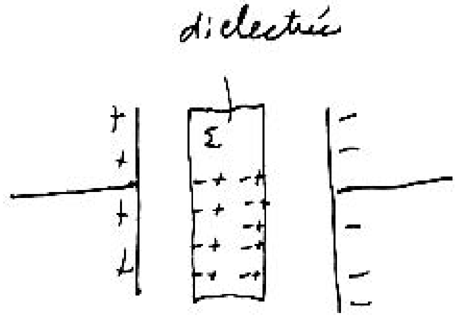
for isotropic media $\boxed{\underset{\sim}{D} = \varepsilon \underset{\sim}{E}}$

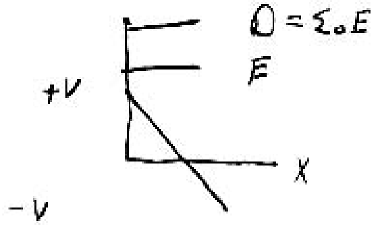
Define electric potential : $\boxed{\underset{\sim}{E} = -\nabla \phi}$

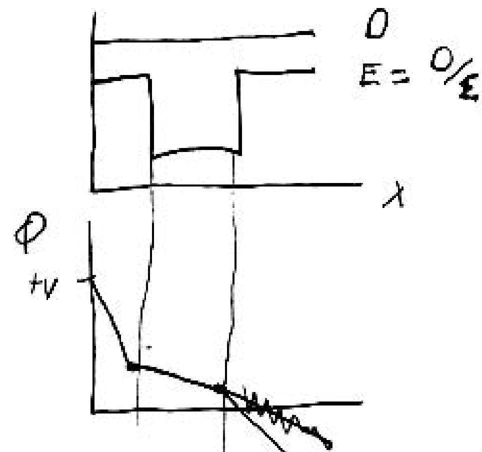
3)

Consider a capacitor!









$$\boxed{E = -\frac{d\Phi}{dx}}$$

Comments of polarization:

$$\underset{\sim}{D} = \varepsilon \underset{\sim}{E}$$

or $\quad \underset{\sim}{D} = \varepsilon_0 \underset{\sim}{E} + \underset{\sim}{P} \qquad$ $\underset{\sim}{P}$ polarization vector

for non isotropic material $\underset{\sim}{P}$ + E can be non parallel

for linear isotropic material

$$\underset{\sim}{P} = \chi_e \varepsilon_0 \underset{\sim}{E} \qquad \chi_e \text{ electric susceptibility}$$

$$\underset{\sim}{D} = \varepsilon_0 (1 + \chi_e) \underset{\sim}{E} = \varepsilon_0 \varepsilon_r \underset{\sim}{E} = \varepsilon \underset{\sim}{E}$$

4).

<u>Current:</u>
<u>2 types</u>

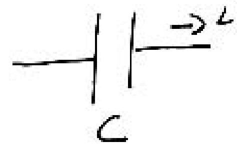
conduction $\boxed{\underset{\sim}{J}_c = \sigma \underset{\sim}{E}}$ just like 

just like flow through a resistor $\boxed{i = V/R}$

$$\underset{\sim}{J}_c = -\sigma \nabla \phi$$

<u>Displacement Current:</u>

$$\boxed{\underset{\sim}{J}_D = \frac{\partial \underset{\sim}{D}}{\partial t}}$$



$\boxed{i_c = C\frac{dV}{dt}}$

time change of voltage

If you have a poor conductor [i.e. lossy dielectric]

Apply $E = E_o e^{j\omega t}$

$$\underset{\sim}{J}_t = \underset{\sim}{J}_c + \underset{\sim}{J}_D = \sigma \underset{\sim}{E} + \frac{\partial}{\partial t}(\varepsilon \underset{\sim}{E}) = \sigma \underset{\sim}{E} + j\omega\varepsilon \underset{\sim}{E}$$

$$\boxed{\frac{J_c}{J_D} = \frac{\sigma}{j\omega\varepsilon}}$$

J_D important at high freq.

<u>Conservation of charge in electrolyte solution</u>:

$$\underbrace{\frac{\partial \rho_f}{\partial t} + (\underset{\sim}{v}\cdot\nabla)\rho_f}_{\frac{D\rho_f}{Dt}} = \underbrace{\nabla\cdot(\sigma\nabla\Phi)}_{-\nabla\cdot J_c} + \underbrace{D\nabla^2\rho_f}_{\text{diffusion}}$$

conservation of conductivity

$$\underbrace{\frac{\partial\sigma}{\partial t}+(\underset{\sim}{v}\cdot\nabla)\sigma}_{\frac{D\sigma}{Dt}} = \underbrace{F^2\omega^2\,\nabla\cdot(\rho_f\nabla\Phi)}_{\text{electro-migration}} + \underbrace{D\nabla^2\sigma}_{\text{diffusion}}$$

$$\begin{cases} F: \text{Faraday's const.} \\ \omega: \text{coeff. of mobility} \\ z: \text{is charge number} \end{cases}$$

assume symmetric electrolyte, like KCl $\quad z_1 = -z_2 = 1$

$$\begin{cases} \rho_f = \sum_{i=1}^{2} z_i F C_i \\ \sigma = \sum_{i=1}^{2} \omega_i z_i^2 F^2 C_i \end{cases}$$

C_i . molar concentration