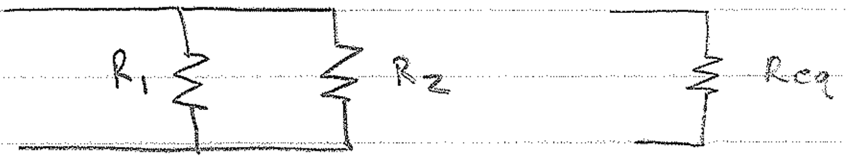
ME 104 HOMEWORK #1 SOLUTIONS

2.3



R_1 = BROWN BLACK RED GOLD $= 10 \times 10^2 \pm 5\% = 1k\Omega \pm 5\%$

R_2 = RED GREEN BROWN GOLD $= 25 \times 10^1 \pm 5\% = 250 \pm 5\%$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ (NOMINAL) $R_{eq} = 200\,\Omega$

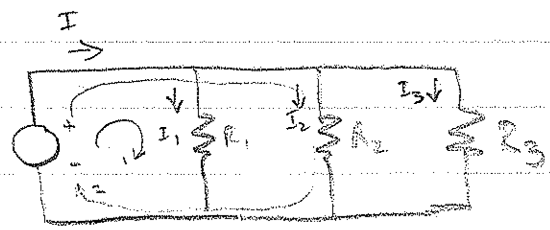
$\frac{1}{R_{eq}-tol} = \frac{1}{R_1(.95)} + \frac{1}{R_2(.95)}$ $R_{eq}-tol = 190\,\Omega$

$\frac{1}{R_{eq}+tol} = \frac{1}{R_1(1.05)} + \frac{1}{R_2(1.05)}$ $R_{eq}+tol = 210\,\Omega$

SO PICK

$R = 200\,\Omega$
tol = 5%
COLOR: a = Red b = BLACK c = BROWN tol = GOLD

2.8



KCL $I = I_1 + I_2 + I_3$

KVL ① $V_S = I_1 R_1$ $V_S = I\,R_{eq}$

② $V_S = I_2 R_1$ $V_S = I\,R_3$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

— CONT. —

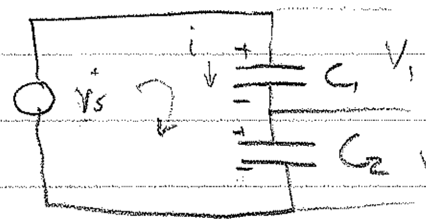
so, $I_1 = \frac{V_S}{R_1} = \frac{I R_{eq}}{R_1} = \frac{R_{eq}}{R_1} I$

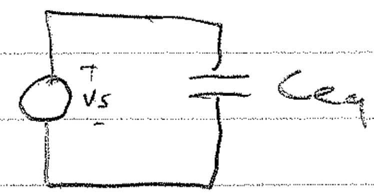
$$\boxed{I_1 = \left(\frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}\right) I}$$

LIKEWISE $\boxed{I_2 = \left(\frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}\right) I}$

$$\boxed{I_3 = \left(\frac{R_1 R_2}{R_1 R_2 + R_2 R_3 + R_1 R_3}\right) I}$$ □

2.10





KVL $\quad V_S - V_1 - V_2 = 0 \qquad \frac{dV_S}{dt} - \frac{dV_1}{dt} - \frac{dV_2}{dt} = 0$

$i = \frac{dV_S}{dt} C_{eq}$

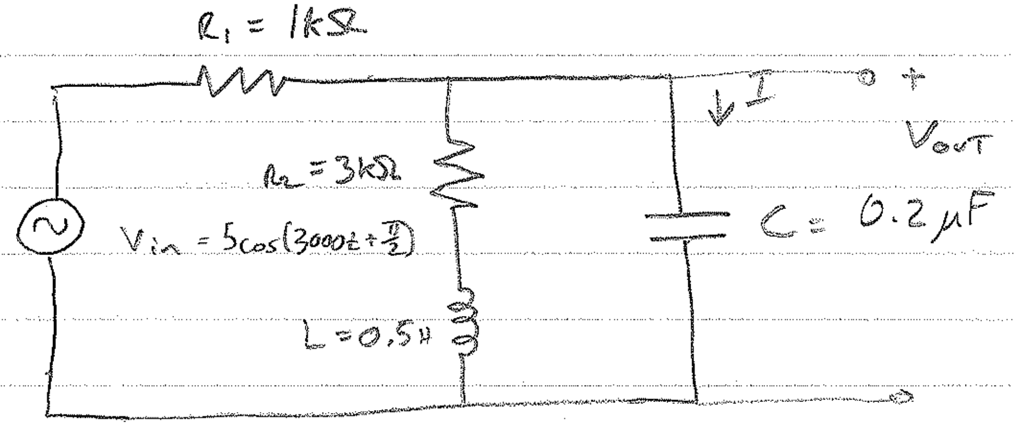
$i = \frac{dV_1}{dt} C_1$

$i = \frac{dV_2}{dt} C_2$

$\Rightarrow \frac{i}{C_{eq}} = \frac{i}{C_1} + \frac{i}{C_2} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\boxed{C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1}{1 + \frac{C_1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}}$$ □

2.27



FROM EXAMPLE 2.7

$$I(t) = 2.22 \angle 159.8^\circ \ \text{mA}$$

AND RECALL $Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{1}{\omega C} \angle -90^\circ$

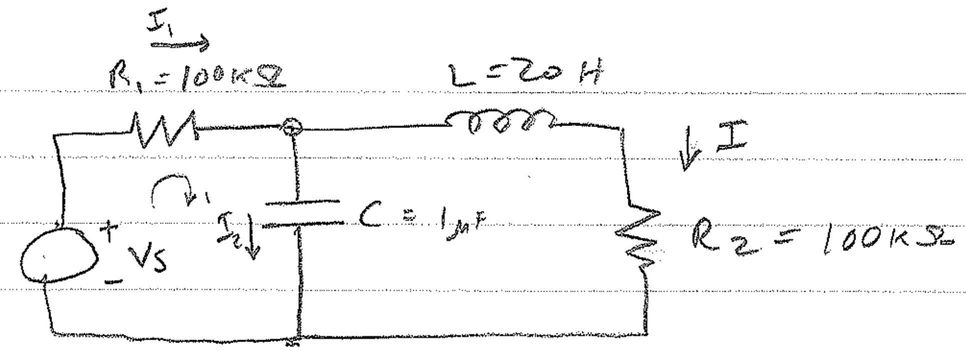
SO $\hat{V}_{OUT} = \hat{I} Z_C = \left(2.22\times10^{-3} \angle 159.8^\circ\right)\left(\frac{1}{\underset{3\times10^{3}}{3000} \cdot \underset{0.2\times10^{-6}}{0.2\,\mu F}} \angle -90\right)$

$$= \left(2.22\times10^{-3} \angle 159.8^\circ\right)\left(1.66\times10^{3} \angle -90\right)$$

$$= 3.69 \angle 69.8^\circ = 3.69 \angle 1.22 \text{ rad}$$

SO $\boxed{V_{out}(t) = 3.69 \cos(3000t + 1.22)\ V}$

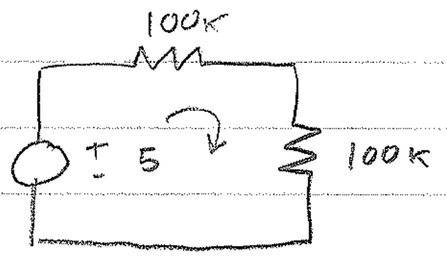
229



FIND $I(t)$

a) $V_S = 5V$ DC

AT DC, THE CIRCUIT BECOMES



$R_{eq} = R_1 + R_2 = 200\,k\Omega$

$$\boxed{I = \frac{V}{R_{eq}} = 0.025\ mA}$$

b) $V_S = 5\cos(\pi t)\ V$

$\hat{V}_S = 5$

KVL

$\circlearrowright_1 : \hat{V}_S = \hat{I}_1 R_1 + \hat{I}_2 Z_C$

OUTER LOOP: $\hat{V}_S = \hat{I}_1 R_1 + \hat{I}(Z_L + R_2)$

KCL

TOP NODE: $\hat{I}_1 = \hat{I}_2 + \hat{I}$

$$\underbrace{\begin{bmatrix} R_1 & Z_C & 0 \\ R_1 & 0 & (Z_L + R_2) \\ 1 & -1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \hat{I} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \hat{V}_S \\ \hat{V}_S \\ 0 \end{bmatrix}}_{b}$$

— CONT —

$$x = A^{-1} b$$

$$\Rightarrow \hat{I} = 2.4398 \times 10^{-5} - 3.8405 \times 10^{-6} j$$
$$= 0.0247 \text{ mA } \angle -0.156 \text{ rad} \quad \text{OR } \angle -8.9^\circ$$

SO $\boxed{I(t) = 0.0247 \cos(\pi t - 0.156)}$

□

5. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$; $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$; $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$

EXPAND TO EVEN & ODD, AND SEPARATE

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} + \frac{x^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

NOW, SUBSTITUTE $x = j\theta$

$$e^{j\theta} = \sum_{n=0}^{\infty} \frac{(j\theta)^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{(j\theta)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{j^{2n}}{(2n)!} \theta^{2n} + \sum_{n=0}^{\infty} \frac{j^{2n+1}}{(2n+1)!} \theta^{2n+1}$$

AND RECALL
$$j^{2n} = (-1)^n \quad \forall n \in \mathbb{Z}$$
$$j^{2n+1} = j(-1)^n \quad \forall n \in \mathbb{Z}$$

SO
$$\boxed{\begin{aligned} e^{j\theta} &= \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \theta^{2n}} + j \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \theta^{2n+1}} \\ e^{j\theta} &= \cos\theta + j \cdot \sin\theta \end{aligned}}$$

□

6. $re^{j\theta} = 1 - 2j$

$$\boxed{\begin{aligned} r &= \sqrt{1^2+2^2} = \sqrt{5} = 2.24 \\ \theta &= \operatorname{atan2}(-2, 1) = -1.11 \text{ rad} = -63.4^\circ \\ &= 5.12 \text{ rad} = 296.5^\circ \end{aligned}}$$

7. $(1+2j)^{-1} = x + jy$

$$\begin{aligned} 1 &= (1+2j)(x+jy) \\ 1 &= x + yj + 2xj - 2y \\ 1 &= x - 2y + (2x+y)j \end{aligned}$$

$$\operatorname{Re}\{1\} = \operatorname{Re}\{x - 2y + (2x+y)j\}$$

$$\Rightarrow 1 = x - 2y$$

$$\operatorname{Im}\{1\} = \operatorname{Im}\{x - 2y + (2x+y)j\}$$

$$\Rightarrow 0 = 2x + y$$

SOLVE $\underbrace{\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{b}$

$$A^{-1}b = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/5 \\ -2/5 \end{bmatrix}$$

So $\boxed{(1+2j)^{-1} = 1/5 - 2/5\, j}$

6/6