HW4 SOLUTIONS

4.2) Fourier Series & fundamental freq (in hertz) of $F(t) = 5\sin(2\pi t)$

fourier Series representation: $\boxed{f(t) = 5\sin(2\pi t)}$

fundamental freq. $f = 1$ hz

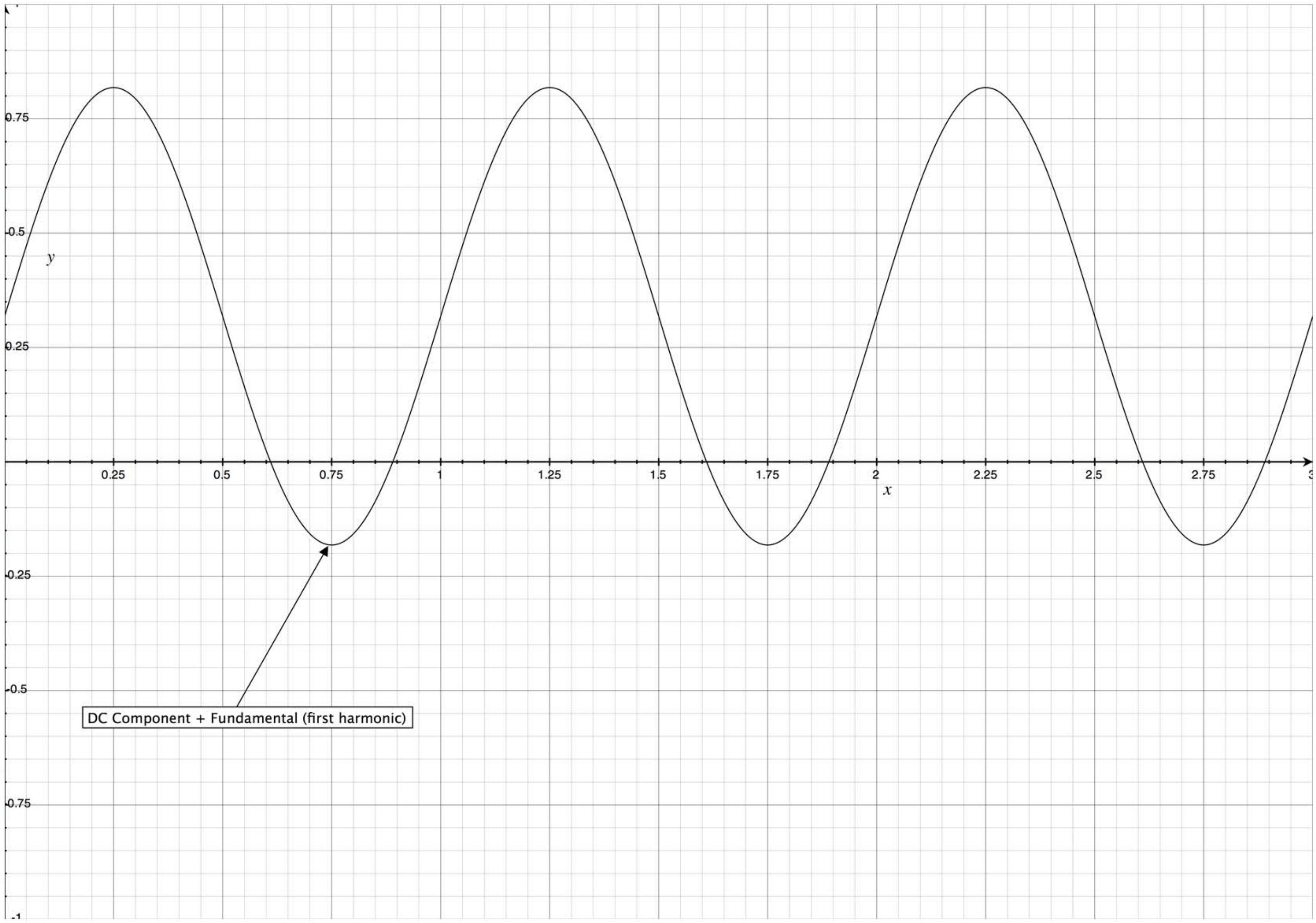
4.5 a) The discrete Fourier Series representation of a half-sin-wave pulse train is mathematically represented by:

$$V(t) = \frac{1}{\pi} + \frac{\sin(2\pi t)}{2} - \frac{2}{\pi}\left[\frac{\cos(4\pi t)}{1\cdot 3} + \frac{\cos(8\pi t)}{3\cdot 5} + \frac{\cos(12\pi t)}{5\cdot 7} + \dots\right]$$

- using a computer plotting application, plot 3 cycles of $V(t)$:

a) DC component + fundamental (first harmonic)

$\omega_0 = 2\pi$ $\quad V(t) = \frac{1}{\pi} + \frac{\sin(2\pi t)}{2}$, $\quad 0 \le t \le 3$

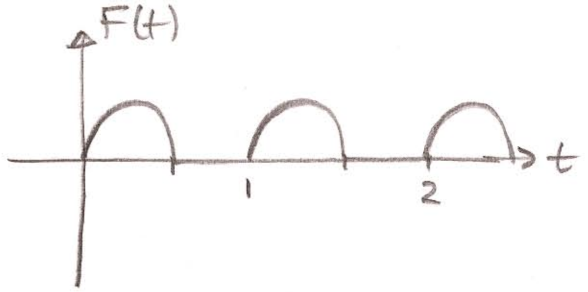


b) DC component + first 10 harmonics (Note that some harmonics have zero amplitude.

$$V(t) = \underbrace{\frac{1}{\pi}}_{\text{DC}} + \underbrace{\frac{\sin(2\pi t)}{2}}_{1^{st}\text{ harmonic}} - \frac{2}{\pi}\Bigg[\underbrace{\frac{\cos(4\pi t)}{1\cdot 3}}_{2^{nd}\text{ harmonic}} + \underbrace{\frac{\cos(8\pi t)}{3.5}}_{4^{th}\text{ harmonic}} +$$

$$\underbrace{\frac{\cos(12\pi t)}{5\cdot 7}}_{6^{th}\text{ harmonic}} + \underbrace{\frac{\cos(16\pi t)}{7\cdot 9}}_{8^{th}\text{ harmonic}} + \underbrace{\frac{\cos(20\pi t)}{9\cdot 11}}_{10^{th}\text{ harmonic}}\Bigg]$$ plot this

<u>Derivation:</u> $F(t) =$ half-sine-wave pulse train, period = 1



use fourier series representation:

$$f(t) = C_0 + \sum_{n=1}^{\infty} \overbrace{f(t)}^{A_n} \cos(n\omega_0 t) + \sum_{n=1}^{\infty} \overbrace{f(t)}^{B_n} \sin(n\omega_0 t)$$

$$C_0 = \frac{1}{T}\int_0^T F(t)\,dt \Rightarrow$$

$$\boxed{F(t) = \begin{cases} \sin(2\pi t), & 0 \le t \le 0.5 \\ 0, & 0.5 \le t \le 1 \end{cases}}$$

$$= \int_0^{0.5} \sin(2\pi t)\,dt = \frac{1}{\pi}$$ via calculator

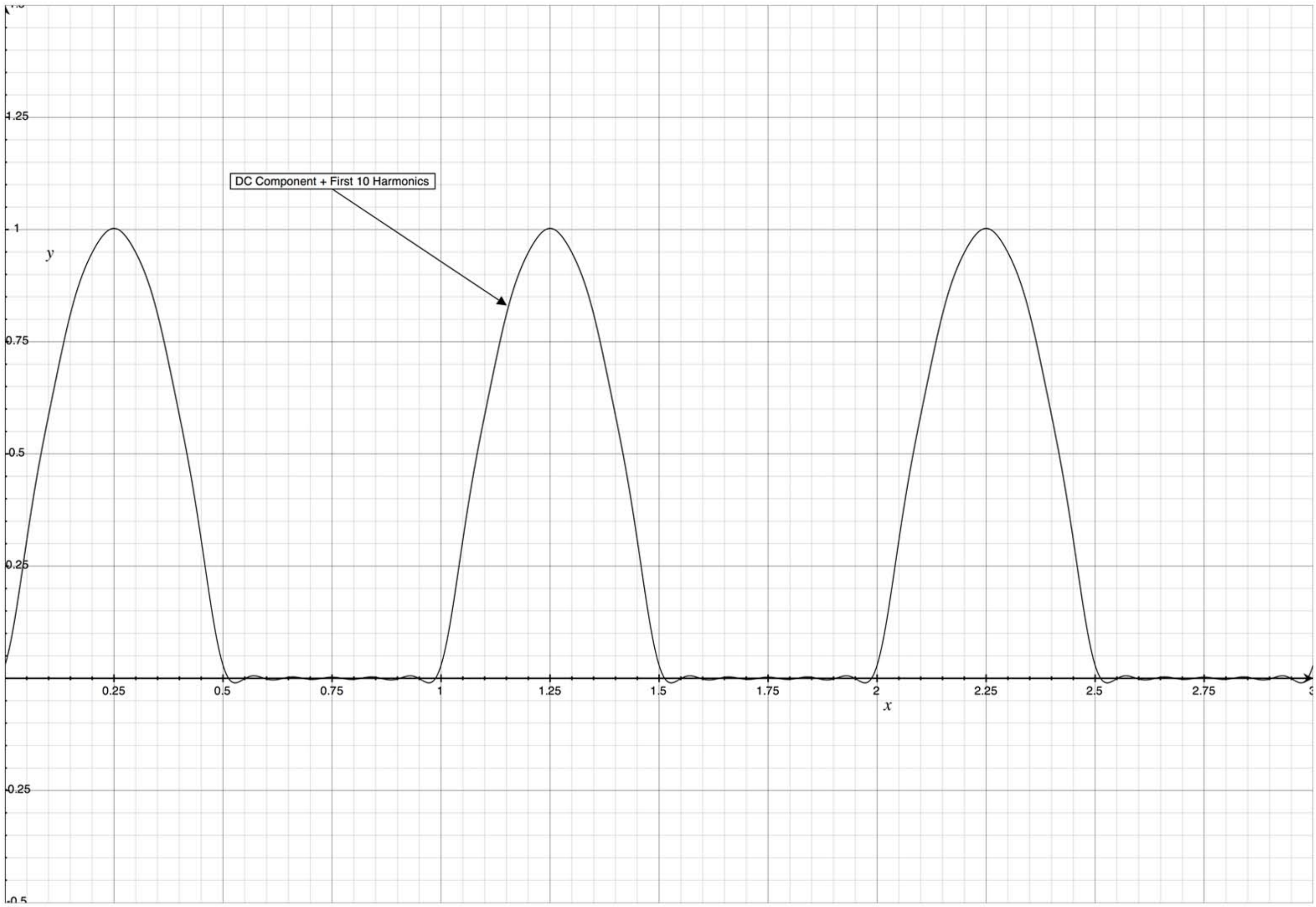
→ DC offset

$$A_n = \int_0^{0.5} \sin(2\pi t)\cos(2\pi n t)\,dt = -\frac{\cos(\pi(n+1))}{2(n+1)\pi} + \frac{\cos(\pi(n-1))}{2(n-1)\pi}$$

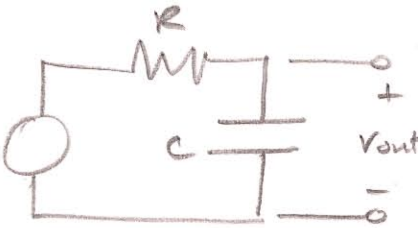
<u>A_n is non-zero for even #s only</u>

$$B_n = \int_0^{0.5} \sin(2\pi t)\sin(2\pi n t)\,dt = \frac{\sin(\pi(n-1))}{2(n-1)\pi} - \frac{\sin(\pi(n+1))}{2(n+1)\pi}$$

<u>B_n is zero for all values of n</u>


DC Component + First 10 Harmonics
y
x
1.25
1
0.75
0.5
0.25
-0.25
-0.5
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3

4.7) $R = 1k\Omega$ $C = 0.01\mu F$



a) what is the bandwidth of the filter?

$$V_{out} = V_{in}\left(\frac{Z_c}{Z_c + R}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R} \cdot \frac{j\omega C}{j\omega C} = \frac{1}{1 + j\omega RC}$$

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$ $\Rightarrow$ cut off frequency is where output power is $1/\sqrt{2}$ input

$$= \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

find $(\omega RC)^2 = 1$

$\omega RC = \sqrt{1}$

$$\boxed{\omega_c = \frac{1}{RC}}$$

thus bandwidth $\Rightarrow$ $\boxed{0 \rightarrow \frac{1}{RC}}$ or $\boxed{0 \rightarrow 100{,}000 \text{ rad.}}$

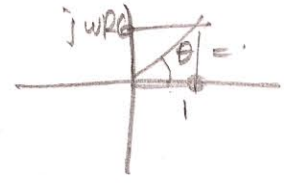
b) if the input is a 2V peak to peak, square wave of T=1 sec what will the filter output be?

Use: $F(t) = \frac{4}{\pi}\sin(\omega_0 t) + \frac{4}{3\pi}\sin(3\omega_0 t) + \frac{4}{5\pi}\sin(5\omega_0 t) + \cdots$ or

$$F(t) = \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi}\sin[(2n-1)\omega_0 t]$$

$\omega_0 = \frac{2\pi}{T} = 2\pi f$ $f = 1$, $\omega_0 = 2\pi$

$$|H(w)| = \left|\frac{V_{out}}{V_{in}}\right| = \left|\frac{1}{1+jwRC}\right| = \frac{\sqrt{1}}{\sqrt{1^2+(wRC)^2}} = \frac{1}{\sqrt{1+w^2R^2C^2}}$$



TOA

$$\angle H(w) = \frac{\angle 1}{\angle(1+jwRC)} = \frac{\angle 0°}{\arctan\left(\frac{wRC}{1}\right)}$$

$$= \angle 0 - \angle \arctan(wRC)$$

$$= -\arctan(wRC) \quad \text{for} \quad \cos(wt - \arctan(wRC))$$

frequency of fourier Series $\boxed{(2n-1)w_0}$ → in radians

all amplitude terms are multiplied by $\left|\frac{V_{out}}{V_{in}}\right|$ @ their respective frequency and each term is phase shifted by $\angle H(w)$ thus:

$$F(t) = \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{1+(2\pi(2n-1))^2(1\times10^{-10})}}\right) \frac{4}{(2n-1)\pi} \sin\left[2\pi(2n-1)t + \frac{\pi}{2} - \arctan(2\pi(2n-1)(1\times10^{5}))\right]$$

Amplitude multiplier

phase shift

plot equation in matlab.
from $t=0$ to $t=2$

use simulink

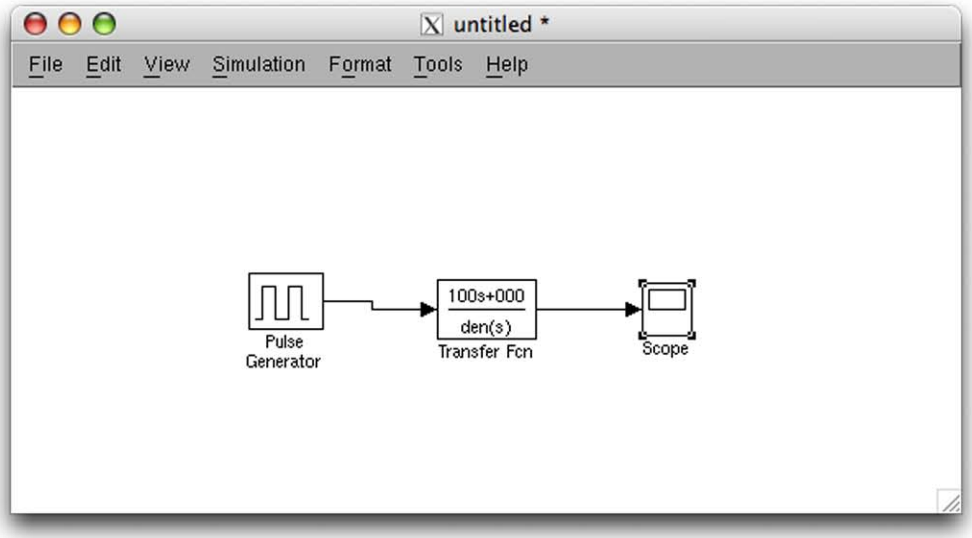
$s \approx jw$

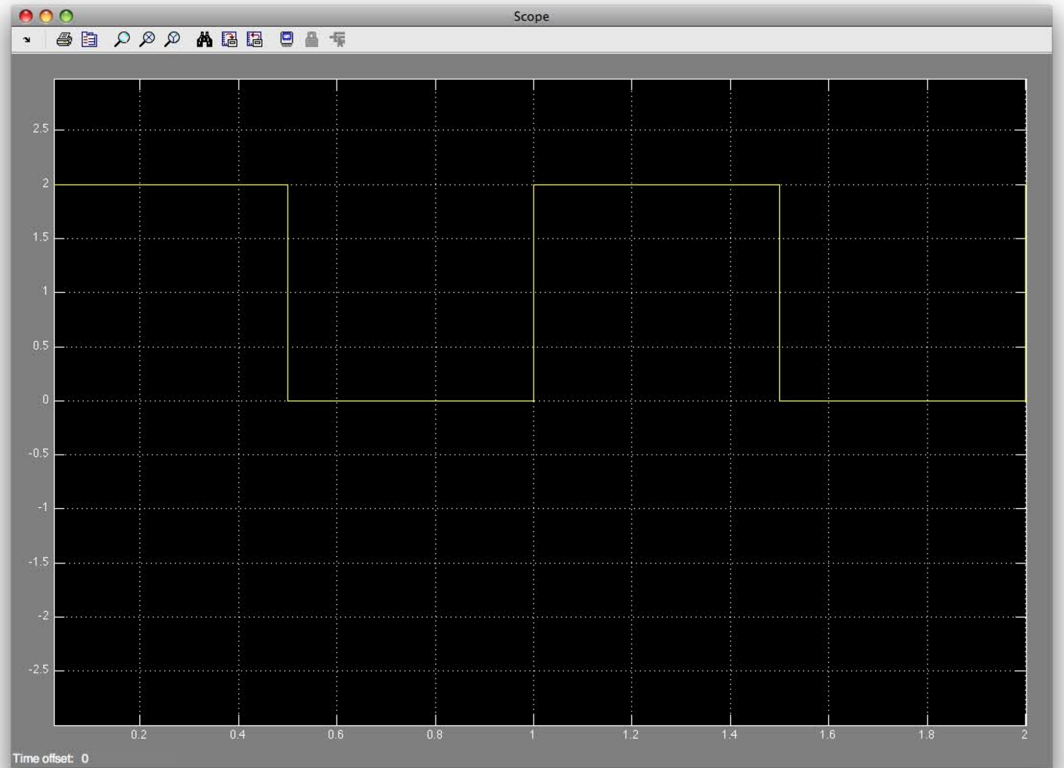
$$\frac{\frac{1}{jwC}}{\frac{1}{jwC}+R} \xrightarrow{s\approx jw} \frac{\frac{1}{sC}}{\frac{1}{sC}+R} \cdot \frac{\frac{s}{R}}{\frac{s}{R}} \longrightarrow \frac{\frac{1}{RC}}{s+\frac{1}{RC}} = \boxed{\frac{100000}{s+100,000}}$$

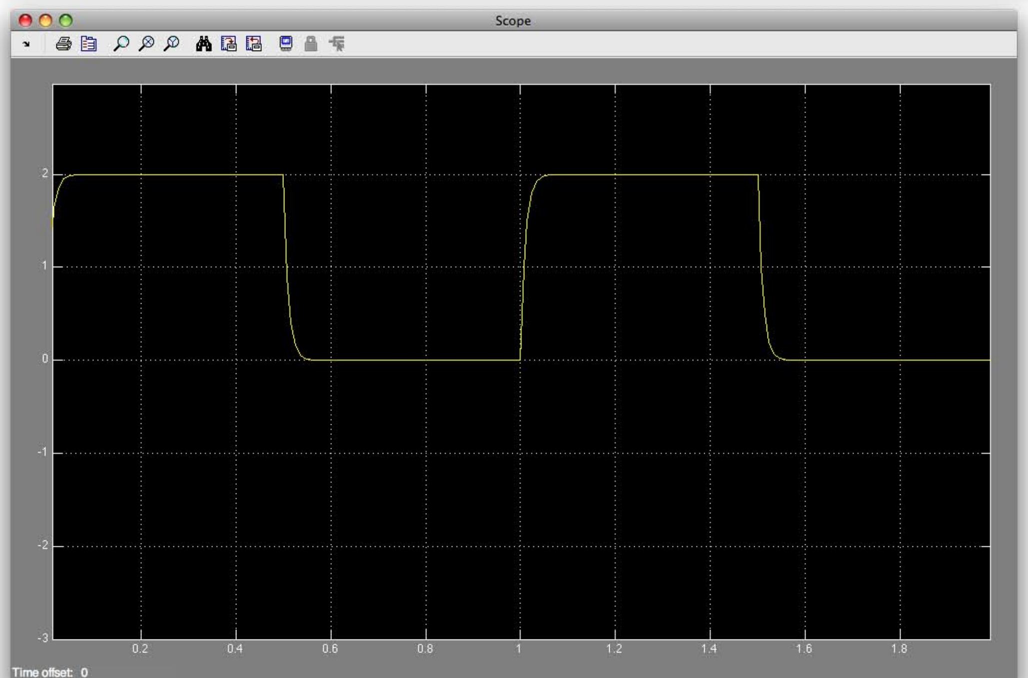
Put in simulink

$1/RC = 100{,}000$, Square wave period $= T = 1$ sec

$$f = \frac{1}{T} = 1\text{ hz} \qquad w_0 = 2\pi f = 2\pi$$


untitled *
File Edit View Simulation Format Tools Help
Pulse Generator
100s+000
den(s)
Transfer Fcn
Scope


Scope
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Time offset: 0


Scope
2
1
0
-1
-2
-3
0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8
Time offset: 0

8.2) Bandwidth = 15 kHz   min sampling rate required?

Min Sampling = 15 kHz × 2   Nyquest thm

$\boxed{= 30\text{ kHz}}$

8.6) 12 bit A/D converter   −5 V to 5 V

how much does the input have to change to be detected?

Analog quantization size: $Q = (V_{max} - V_{min})/2^N$

$Q = (5+5)/2^{12} = 0.002441\text{ V}$

input has to change by at least 0.002441 Volts to be detected