ME104 Midterm I
1 hour and 10 minutes
Closed book & notes, no calculators.

Problem 0. What is your name? Solutions.

Problem 1. Compute the magnitude and phase of $\frac{1+j}{1-j}$.

$$\frac{1+j}{1-j}\left(\frac{1+j}{1+j}\right) = \frac{2j}{2} = j$$

Magnitude = 1

Phase = + 90°

$= 1\angle 90^\circ$

Problem 2. Find the phasor representing

a) $\cos(\omega t)$

b) $4\sin(3t+\frac{\pi}{4})$

a) $\cos wt = 1\angle 0^\circ$

b) $4\sin(3t+\pi/4) = 4\cos\left(\frac{\pi}{2} - 3t - \frac{\pi}{4}\right)$

$= 4\cos\left(\frac{\pi}{4} - 3t\right) = 4\cos(3t - \pi/4)$

$= 4\angle -\pi/4 = 4\angle -45^\circ = 4\angle -45^\circ$

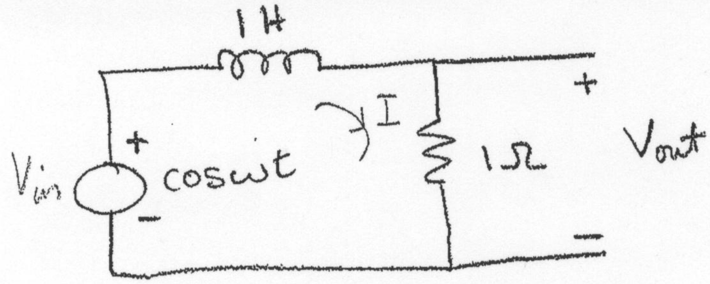
Problem 3. Find the sinusoid represented by the phasor $-j$.

$-j = -1\angle 90^\circ$

$= -1\cos(wt + 90^\circ)$

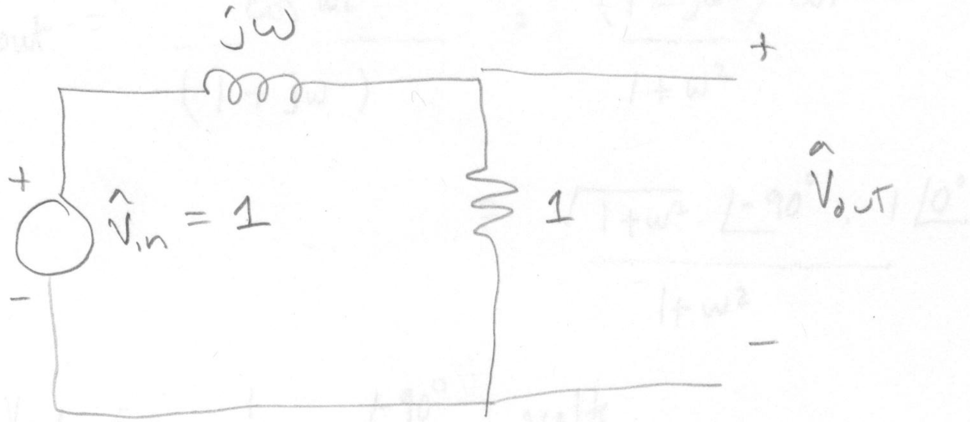
$= \sin wt$

Problem 4. Find sinusoidal steady state of Vout in the circuit below.



Step # 1 Convert to phasors

Step # 2 $\frac{d}{dt} \rightarrow j\omega$ (insert complex impedances)



Step # 3 Solve

using impedance divider formula

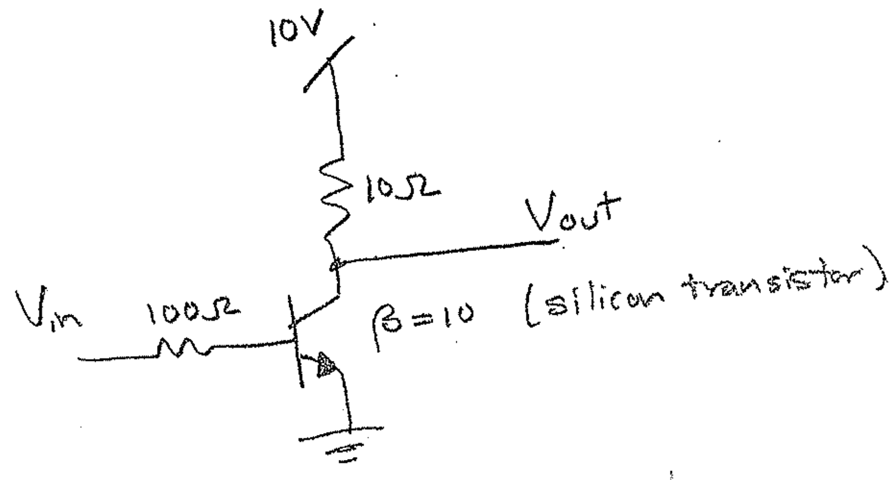
$$\hat{V}_{out} = \frac{1}{1 + j\omega} \cdot \hat{V}_{in} = \frac{1}{1+j\omega}$$

step # 4 convert to sinusoids

$$V_{out}(t) = |\hat{V}_{out}| \cos(\omega t + \angle \hat{V}_{out})$$

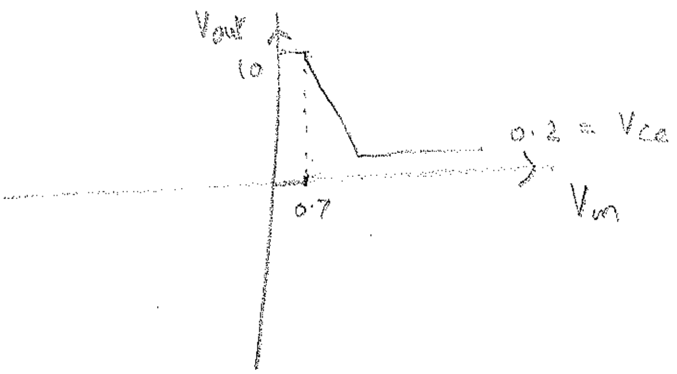
$$= \frac{1}{\sqrt{1+\omega^2}} \cos(\omega t - \operatorname{atan}(\omega))$$

Problem 5. Plot Vout versus Vin for the circuit below.



$V_{in} = I_B(100) + V_{be} = 0.7 + 100 I_b$

$V_{out} = 10 - \beta I_b(10) = 10 - 100 I_b$



when $V_{in} > V_{be} = 0.7$,
transistor conducts
∴ $V_{out} = V_{ce} \approx 0.2$

when $V_{in} < V_{be}$, Tx is in
cut off.
∴ $V_{out} = 10V$

6. Show that the mechanical impedance of a mass is $-\omega^2 m$.

We know for mass m,

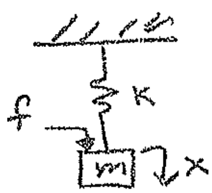
$$f = m \cdot \frac{d^2x}{dt^2}$$

Substitute $\frac{d}{dt} \rightarrow j\omega$

$$\therefore f = m(j\omega)^2 \cdot x = -m\omega^2 x$$

$$\text{Impedance} = f/x = \underline{-m\omega^2}$$

7. What is the mechanical impedance $\frac{\hat{f}}{\hat{x}}$ for the system below.



$$f = m \cdot \frac{d^2x}{dt^2} + kx$$

$$f = m(j\omega)^2 \cdot x + kx$$

$$-m\omega^2 x + Kx.$$

$$\therefore \frac{f}{x} = \underline{-m\omega^2 + K.}$$