On Trajectory Generation for Flexible Robots

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Received November 17, 1986; accepted November 25, 1986

A trajectory based on a Gaussian velocity profile is presented as an alternative to the double square pulse acceleration profile. The new trajectory leads to the fast positioning of the tip of a flexible robot with a minimal excitation of high-frequency modes. The torques necessary to move the robot according to this trajectory show a very smooth behavior. The absence of high-frequency content, present when double square pulse accelerations are considered, eliminates the occurrence of undesired residual vibrations produced by modeling uncertanties at high frequencies. The excellent results obtained suggest the use of this new trajectory for fast and precise positioning of flexible robots.

二重平方パルス加速度曲線に変わるものとしてガウス速度曲線に基ずく軌道を提案する．新しい軌道は高周波数モードの励起を最小にするフレキシブル・ロボットの先端の高速位置決めを可能にする．この軌道によってロボットを駆動するのに必要なトルクは非常に滑らかな振る舞いを見せる．二重平方パルス加速度曲線の場合に現われるような高周波成分を持たないことは、高周波帯におけるモデル化の不確かさによって引き起こされる望ましくない残留振動の発生を防ぐ．ここで得られた優れた結果はフレキシブル・ロボットの高速・高精度位置決めにこの新しい軌道を用いることができることを意味している．

I. INTRODUCTION

It is widely accepted that bang-bang acceleration profiles lead to optimal trajectories for the case of rigid body robots. However, the same acceleration profiles do not lead to good behavior in the case of flexible robots, as the high-frequency content inherent in the double pulse produces perturbations in the general response of the flexible system, which in essence are very difficult if not impossible to control and lead to undesired tip oscillations.

In a recent work,[1] a finite element technique has been introduced to compute the torque that must be applied at one end of a flexible link to produce a desired motion at the other end. Figure 1 shows a typical one-link flexible manipulator. The desired

Journal of Robotic Systems, **4**(2) 229—235 (1987)
 CCC 0471-2223/87/020229-07$04.00

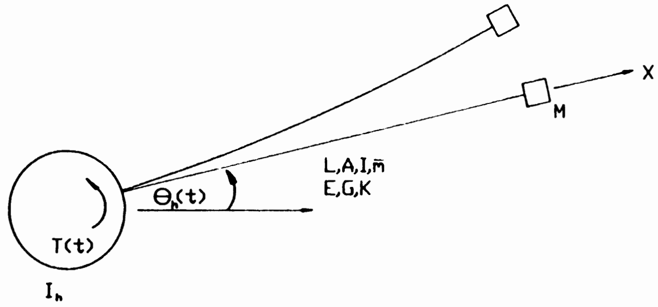

Figure 1. Deformation in a flexible arm.

tip acceleration, velocity, and displacements are depicted in Figures 2a, 2b, and 2c, respectively. The corresponding torque is calculated and shown in Figure 2d. As can be seen, the torque contains high-frequency spikes produced by the coupling between the high-frequency content of the double pulse and the high-frequency modes of the flexible system.

Although the motor may be driven so that it can provide the required torque, the necessary updating time may be so small that the electrical system could be brought into resonance with the mechanical system, thereby producing perturbations in the overall behavior. Modeling uncertainties in both the mechanical and electrical subsystems tend to be larger at high frequencies, and they will have significant effect when bang-bang control is attempted. In what follows, a new tip trajectory is proposed that allows the torques to vary uniformly without high-frequency perturbations. This leads to a smooth behavior of the tip motion and avoidance of spurious vibrations due to system sensitiveness.

II. ACTUATING TORQUES FOR GIVEN TIP MOTIONS

Reference 1 describes a technique based on the finite element method to compute the torques that produce desired tip motions in a one-link flexible robot. The procedure obtains (using the finite element equations) the complex transfer functions relating the tip motion with the actuating torque.

The finite element equations in the frequency domain for the system shown in Figure 1 are [1]:

$$\left[\mathbf{K} - \frac{1}{\omega^2}\mathbf{M} + \frac{1}{i\omega}\mathbf{C}\right]\hat{\ddot{\mathbf{v}}}(\omega) = \mathbf{I}\,\hat{T}_h(\omega) \tag{1}$$

where **M, C,** and **K** are the mass, damping, and stiffness matrices, respectively, **I** is a vector containing a unit value at the degree of freedom corresponding to the hub and zero in the rest, $\ddot{\mathbf{v}}$ is the acceleration vector, and T_h is the torque at the hub. The symbol ˆ stands for Fourier transform.

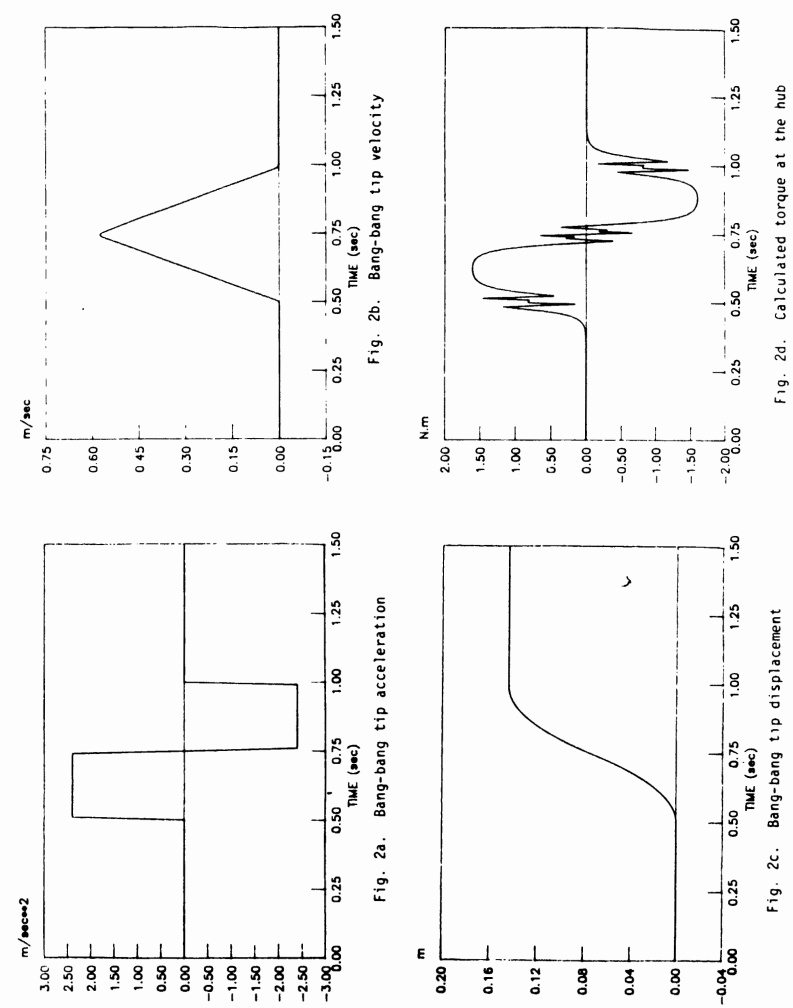

Figure 2. Bang-bang tip trajectory. (a) Bang-bang tip acceleration. (b) Bang-bang tip velocity. (c) Bang-bang tip displacement. (d) Calculated torque at the hub.

From (1)

$$\hat{\ddot{\mathbf{v}}}(\omega) = \left[\mathbf{K} - \frac{1}{\omega^2}\mathbf{M} + \frac{1}{i\omega}\mathbf{C}\right]^{-1} \mathbf{1}\,\hat{T}_h(\omega) = \mathbf{G}\,\hat{T}_h(\omega). \tag{2}$$

A direct relation between acceleration at the tip and the actuating torque may be obtained from Eq. (2) as follows:

$$\hat{\ddot{v}}_t(\omega) = G_{th}\,\hat{T}_h(\omega) \tag{3}$$

where $\ddot{v}_t$ is one (the last) component of the vector $\ddot{\mathbf{v}}$. Given a tip acceleration profile $\ddot{v}_t(t)$, Eq. (3) can be solved by means of the fast Fourier transform to compute $T_h(t)$.

III. TRAJECTORY GENERATION

The torque-to-tip transfer function $G_{th}(\omega)$ in Eq. (3) is invertible for all ω (this is equivalent to the tip not being a node for any vibrational mode), and so the tip can theoretically be moved along any Fourier-transformable trajectory that starts at $v_t = \alpha$ and ends at $v_t = \beta$. If $v_t(t)$ is the desired trajectory, the torque may be readily obtained by solving

$$\hat{T}_h(\omega) = G_{th}^{-1}\,\hat{\ddot{v}}_t(\omega) \tag{4}$$

(see Fig. 2). However, due to modeling errors, G_{th} tends to be imprecisely known at high frequencies. To understand the effects of this uncertainty in G_{th}, let $\overline{G}_{th}$ be the true value of the complex transfer function. Then the application of the torque $G_{th}^{-1}\hat{\ddot{v}}_t$ at the hub results in the tip acceleration

$$\hat{\ddot{v}}_{\text{true}} = \overline{G}_{th}(G_{th}^{-1})\hat{\ddot{v}}_t. \tag{5}$$

The resulting tip acceleration error, $\ddot{e}$, is given by

$$\hat{\ddot{e}} = (I - \overline{G}_{th}\,G_{th}^{-1})\hat{\ddot{v}}_t. \tag{6}$$

From Eq. (6) we see that if $G_{th} \neq G_{th}$ or, more precisely, if $(I - G_{th}\,G_{th}^{-1})$ is large for large ω, then the acceleration error may be large if $\hat{\ddot{v}}_t(\omega)$ has significant spectral content at high frequencies. Furthermore, as mentioned earlier in this paper, it is difficult for a motor to provide very high-frequency torque since electrical time constants become important at high frequencies. These effects will cause unwanted residual vibrations in the link after the tip reaches the desired position.

The objective is therefore to find a trajectory which moves the tip to the desired position quickly, and at the same time has minimal frequency bandwidth. These are two competing objectives which can be conveniently made quantitative when expressed in terms of the tip velocity profile. The condition that the motion be quick can be expressed as a requirement that the region of time where the velocity is nonzero is

concentrated near a point (take this point to be 0). The RMS "time-bandwidth," τ, of $\dot{v}_t$ is a suitable measure of this:

$$\tau^2 \triangleq \frac{\int_{-\infty}^{\infty} \dot{v}_t^2(t)t^2dt}{\int_{-\infty}^{\infty} \dot{v}_t^2(t)dt}. \tag{7}$$

Similarly,

$$\sigma^2 \triangleq \frac{\int_{-\infty}^{\infty} \hat{\dot{v}}_t^2(\omega)\omega^2d\omega}{\int_{-\infty}^{\infty} \hat{\dot{v}}_t^2(\omega)d\omega} \tag{8}$$

where σ is the RMS frequency-bandwidth of $\dot{v}_t$ and measures roughly how much high-frequency content there is in the trajectory. The advantage of these measures is that the tradeoff between speed and frequency content can be expressed precisely in terms of the uncertainty principle[2] as:

$$\sigma\tau \geq \frac{1}{2} \tag{9}$$

Thus, the best we can hope to do is choose, say, τ and then select the tip velocity profile such that equality is obtained in Eq. (9). Given τ, the velocity profile which attains the lower bound is

$$\dot{v}_t = \frac{\beta - \alpha}{\sqrt{2\pi\tau^2}}\,e^{-t^2/2\tau^2}. \tag{10}$$

This is the Gaussian velocity profile and is shown together with the corresponding position and acceleration trajectories in Figures 3b, 3c, and 3a, respectively.

We can therefore generate the hub torque as follows. Given $\beta - \alpha$, choose τ and therefore $\dot{v}_t(t)$ by (10). Here τ controls the time scaling of the Gaussian and therefore the speed. In practice, we must truncate the tails of the Gaussian. (Somewhere between 3 and 4τ should be sufficient.) In the example shown in Figure 3, $\tau = 0.1$. The resulting acceleration profile is

$$\ddot{v}_t(t) = \frac{-t(\beta - \alpha)}{\sqrt{2\pi\tau^3}}\,e^{-t^2/2\tau^2}. \tag{11}$$

Finally we solve for the torque according to Eq. (4) and the inverse fast Fourier transform.

Figure 3d shows the torque necessary to move the flexible manipulator according

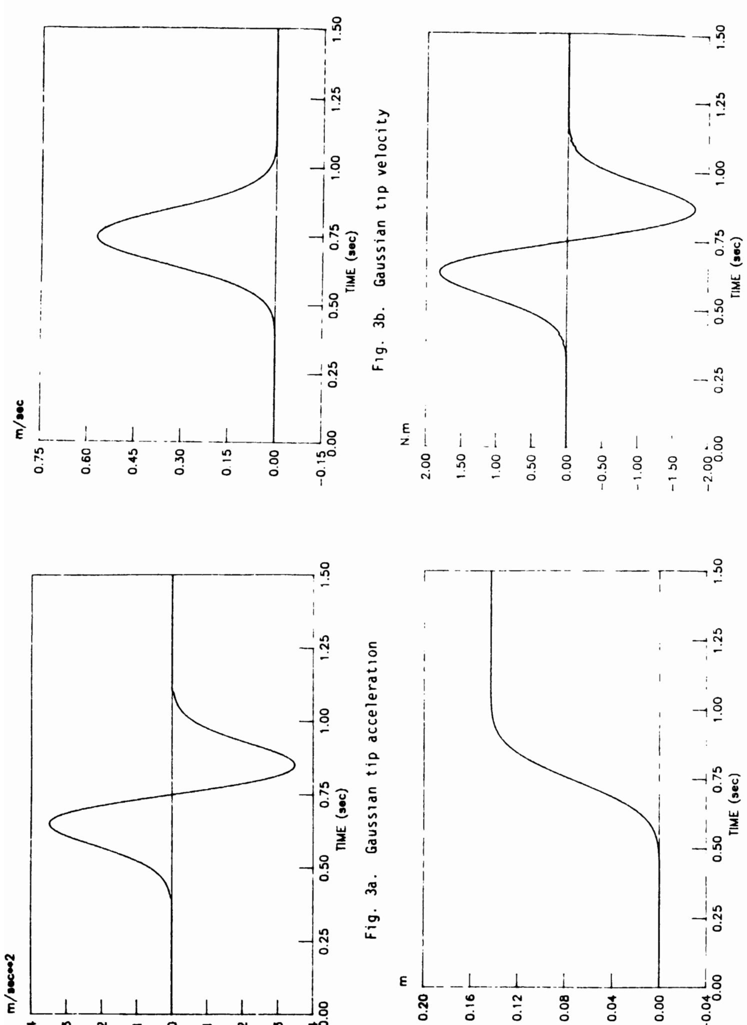

Fig. 3a. Gaussian tip acceleration
Fig. 3b. Gaussian tip velocity
Fig. 3c. Gaussian tip displacement
Fig. 3d. Calculated torque at the hub

Figure 3. Gaussian tip trajectory. (a) Gaussian tip acceleration. (b) Gaussian tip velocity. (c) Gaussian tip displacement. (d) Calculated torque at the hub.

to the Gaussian trajectory defined in Figure 3c. It can be seen how the high-frequency content has disappeared except for a minimal noise of short duration at the beginning and end of the torque profile. This torque needs to be applied for a little larger duration than that corresponding to the double square pulse trajectory. This is the small price to be paid for much better performance.

IV. CONCLUSIONS AND ACKNOWLEDGMENTS

Tip trajectories based on bang-bang acceleration profiles are required for time optimal control in the case of rigid robots. These, however, produce high-frequency perturbations in flexible robots, which are difficult to control and may cause oscillatory behavior of the tip motions of the flexible links. An alternative tip trajectory has been presented based on a Gaussian velocity profile that eliminates the high-frequency content of the response and leads to a very smooth behavior of the flexible system.

It is then concluded that this type of trajectory should be included, rather than the double square pulse, for the end positioning of flexible robots.

The authors want to acknowledge the support of this work by the National Science Foundation under Grant No. 8421415.

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