Partial Phase Synchronization of Uncoupled Populations: An Application of Phase Reduction Methods

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Abstract

We show that populations of identical uncoupled neurons exhibit partial phase synchronization when stimulated with independent, random unidirectional current spikes with interspike time intervals drawn from a Poisson distribution. We characterize this partial synchronization using the probability distribution function of the phases for the population, and consider an analytical approximation and numerical simulations of phase models and full conductance-based models of typical Type I (Hindmarsh-Rose) and Type II (Hodgkin-Huxley) neurons. We show quantitatively how the extent of the synchronization depends on the magnitude and mean interspike frequency of the stimulus. Good agreement is found between the analytical approximation and the numerical simulations.

Models

- Population represented by 1000 uncoupled neurons
- Neurons represented by typical space-clamp conductance-based ODE systems
  $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$, $\quad \mathbf{x} = (V, m)^T$, Voltage $\quad V \in \mathbb{R}$, Gating Variabels $\quad m \in \mathbb{R}^{n-1}_{[0,1]}$
  Type I: Hindmarsh-Rose
  Type II: Hodgkin-Huxley
- Assume baseline current I_b values that yield a stable limit cycle
  Hindmarsh-Rose $I_b = 5mA$
  Hodgkin-Huxley $I_b = 10mA$
- Subjected to uncorrelated random depolarizing spike trains
  Interspike intervals drawn from a Poisson distribution:

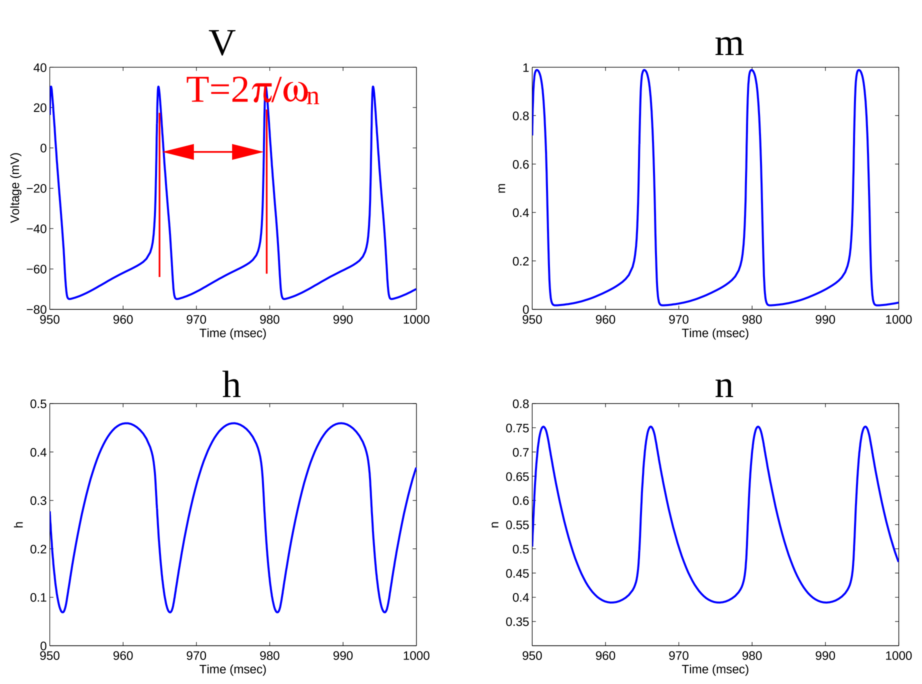
$$p(\tau; \alpha) = \alpha e^{-\alpha\tau}$$

$$I(t) = \bar{I} \sum_k \delta(t - \tau_k), \quad I_{avg} = \alpha \bar{I}$$

We scale $\bar{I} \approx 1mA$ generically, and following [3] we take $\alpha \approx 83$ Hz.

Periodic Dynamics

$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$ has a stable limit cycle $\mathbf{x}^\gamma(t) : \mathbf{x}^\gamma(t_0) = \mathbf{x}^\gamma(t_0 + T)$, $\quad T \equiv$ Period



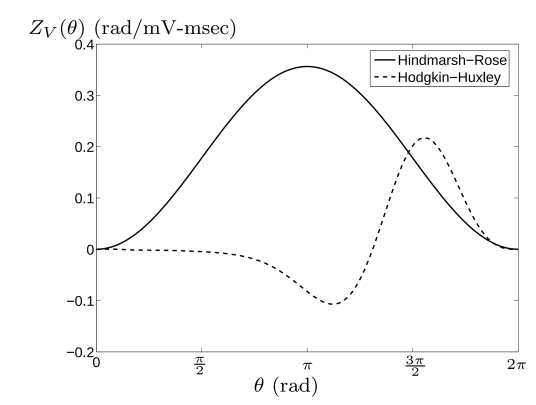
Example time series of a Hodgkin-Huxley neuron model exhibiting periodic behavior.

Phase Reduction

$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) \Rightarrow \dot{\theta} = \omega_n + \frac{Z_V(\theta)}{C} I(t)$, following [1, 2].

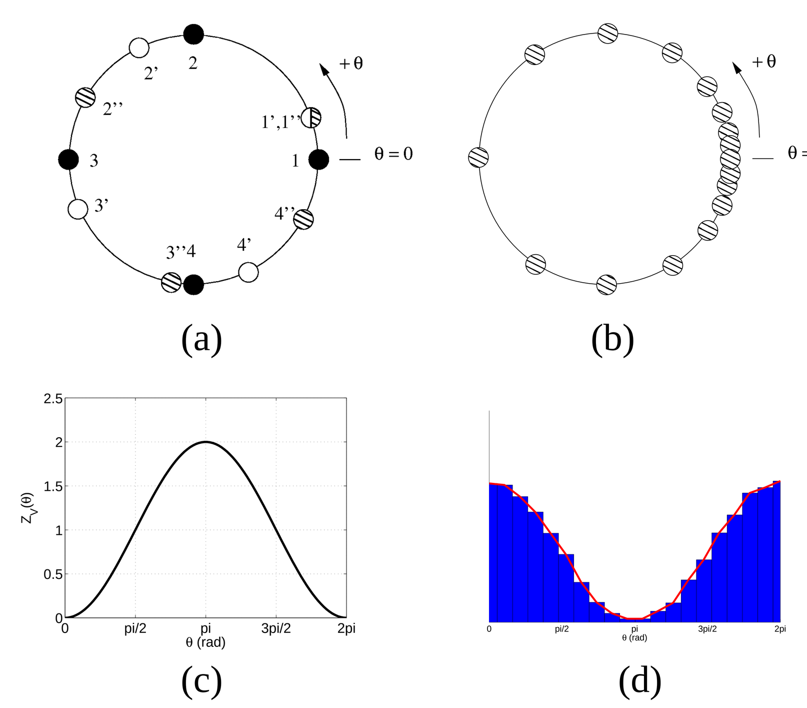
Introduce phase variable $\theta(\mathbf{x}) : \mathbb{R}^n \to [0, 2\pi)$ such that, in the absence of input $I(t)$, we have $\dot{\theta} = \omega_n$. Let $\theta = 0$ correspond to a marker event - the peak value of V on periodic orbit $\mathbf{x}^\gamma(t)$. With input, $\dot{\theta} = \omega_n + \frac{Z_V(\theta)}{C} I(t)$, where we compute the Phase Respone Curve $Z(\theta)$ by solving the appropriate adjoint equation using XPPAUT.

Phase Response Curves



Typical phase response curves for the neuron models we consider.

Intuitive Description

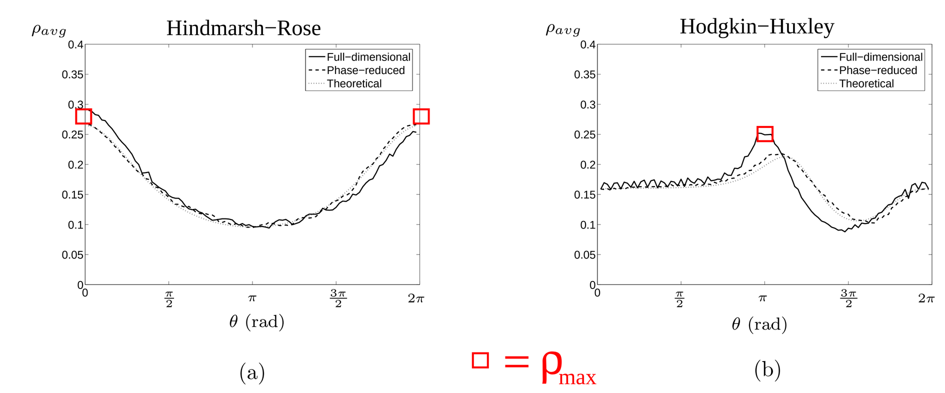


Imagine a population of four simple oscillators with $Z(\theta) = 1 - \cos(\theta)$, as in (c), starting with positions $i \in \{1 \dots 4\}$ shown in plot (a). These oscillators drift under ω_n to positions i'. If all receive an identical "average" stimulus, they move further to positions i'' due to the $Z(\theta)$.
Figure (b) illustrates the process in (a) applied to many oscillators results in a "bunching" around $\theta = 0$. Taking a histogram, we get (d), an approximation to the phase distribution.

Simplifying the density evolution equation $\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial \theta}[(\omega + Z(\theta)I(t)/C)\rho(\theta, t)]$ by assuming negligible diffusion yields an approximation of the steady-state distribution

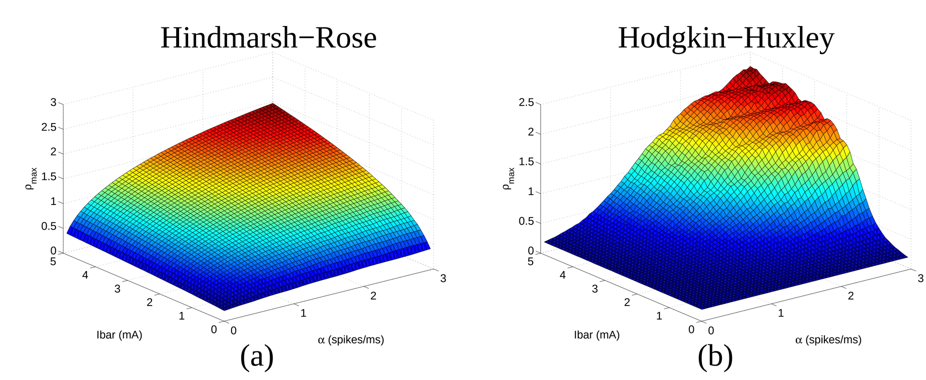
$$\rho_s(\theta) \approx \frac{c}{\omega + Z_V(\theta) I_{avg}} \quad , \quad \int_0^{2\pi} \rho_s(\theta) d\theta = 1 \quad .$$

Phase Density Evolution



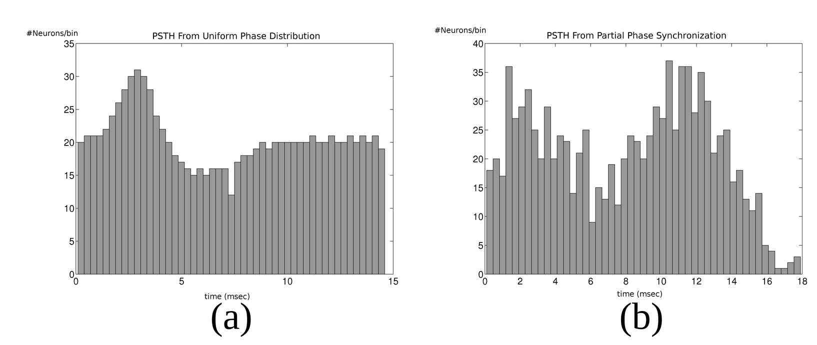
Comparison of the long-time phase density distributions as calculated from simulations of the phase-reduced models, simulations of conductance-based models, and the predictions of the simplified density evolution equation.

Numerical Results - Maximum Phase Synchronization



Using the phase reduced model, it becomes feasible to sweep large regions of parameter space. Here we see how the mean interspike frequency, α, and spike magnitude, $\bar{I}$, affect the peak of the density distribution - a measure of synchrony.

Implications to Population Dynamics



Example peri-stimulus time histograms showing that a population of Hodgkin-Huxley neurons reacts differently to a step input when partially phase synchronized (b) compared to uniformly distributed (a).

References

[1] E. Brown, J. Moehlis, and P. Holmes. On the phase reduction and response dynamics of neural oscillator populations. *Neural Comp.*, 16:673–715, 2004.

[2] P. W. Danzl, J. Moehlis, and G. Bonnet. Partial phase synchronization of neural populations due to random Poisson inputs. 2007. Submitted to *J. Comp. Neurosci.*

[3] W.R. Softky and C. Koch. The highly irregular firing of cortical cells is inconsistent with temporal integration of random EPSPs. *J. Neurosci.*, 13(1):334–350, 1993.