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A Mechanistic Model of Separation Bubble

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joint work with **J.E. Marsden** (Caltech) and **H.M. Nagib** (IIT)

California Institute of Technology

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- **Methodology**: use of analogies with other physical phenomena and basic mechanical/dynamical systems principles.
- **Outcome**:
  - explanation of the nature of observed hysteresis;
  - suggestion of a number of non-trivial questions to be answered experimentally;
  - model based on intuitive physical variables.

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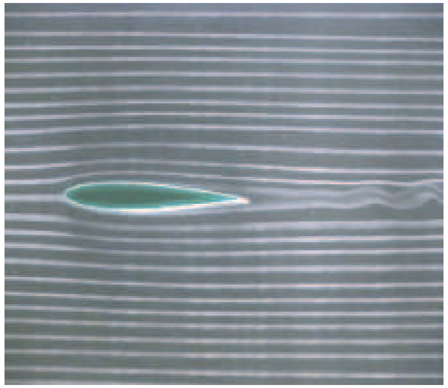

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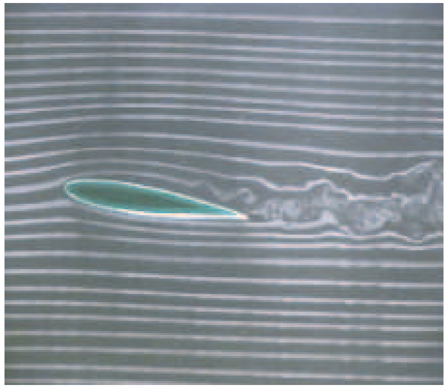

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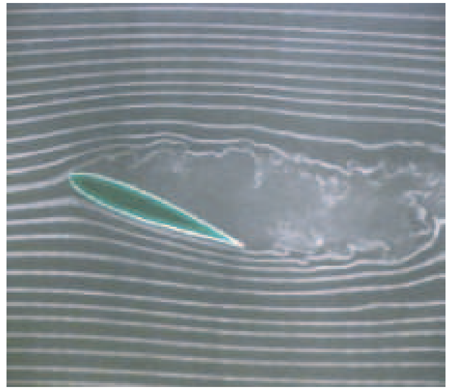

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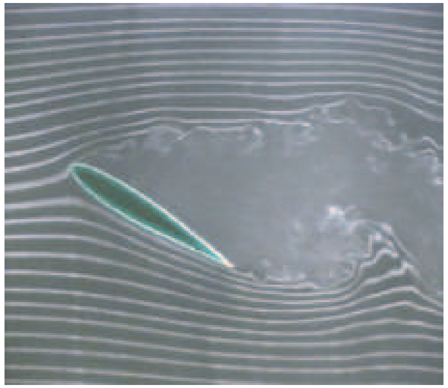

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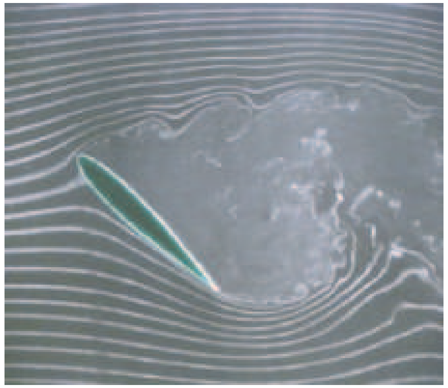

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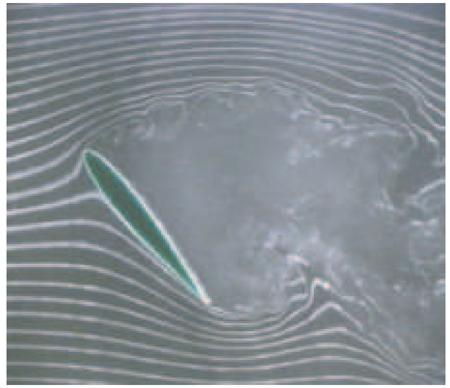

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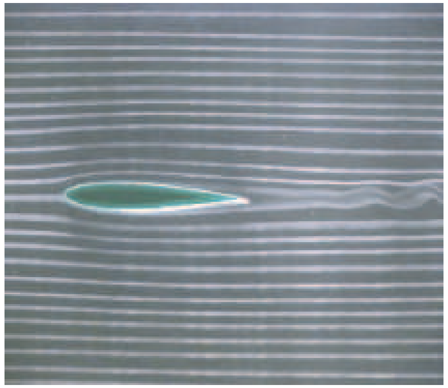

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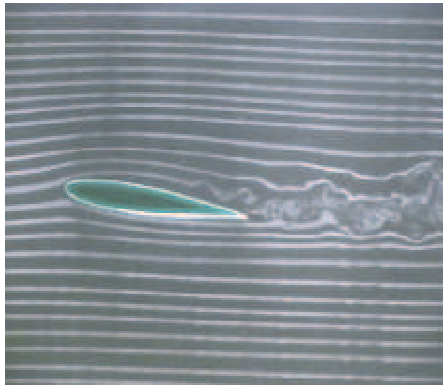

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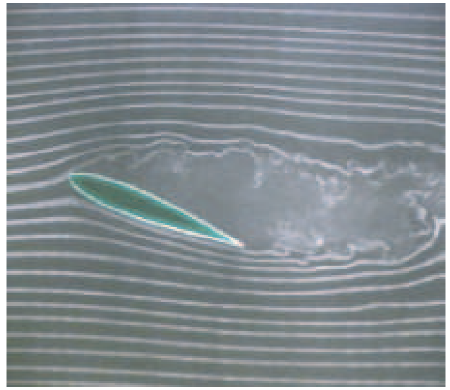

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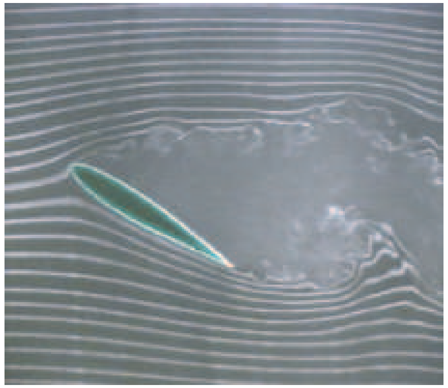

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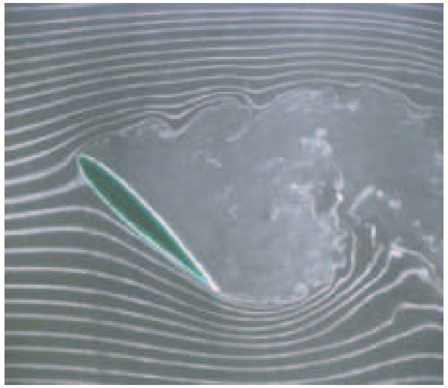

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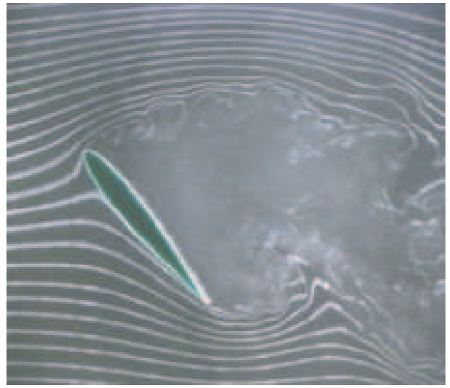

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Historical remark: term “separation bubble” is due to Jones (1933).

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- Why feedback control?: It is more efficient and reliable versus open loop control based on actuator operating schedule.
- How to control?: Via ***model-based observer***, which should be
  - *low-dim*, for computational efficiency in real flight;
  - *physically motivated*, to reflect actual behavior.

Closed-loop dynamic control system

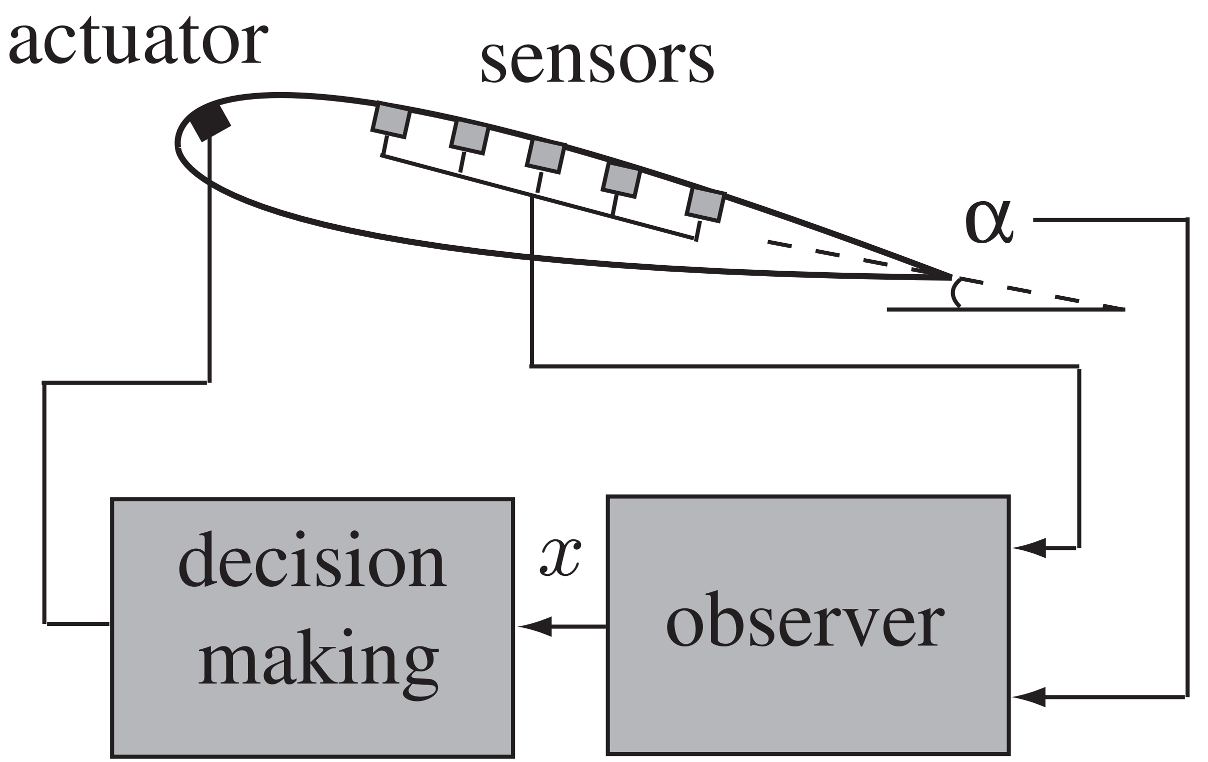

Figure: **Schematics**

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Classical example of the successful phenomenology: Landau equation (Landau, 1944; Stuart, 1960):

$$\frac{\mathrm{d}A}{\mathrm{d}t} = A - \gamma A \left| A \right|^2 .$$

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- Physical variables:

  (i) lift Z

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$$B_{tt} = -k_1 B_t + k_2 \left[B_s(\alpha) - B\right],$$

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Question: is this linear model adequate?

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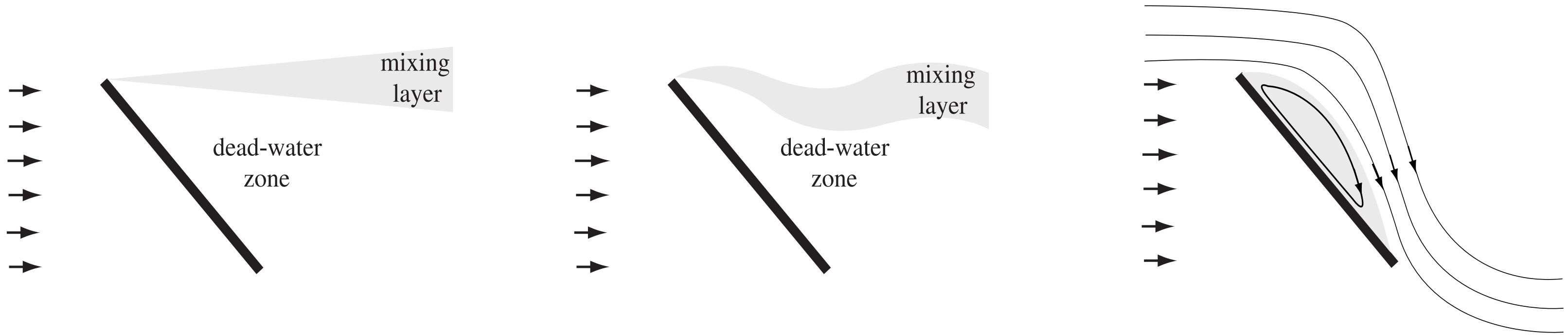
- *Mechanism*: the excitation (vs. forcing) generates Large Coherent Structures transferring high momentum fluid towards the surface:



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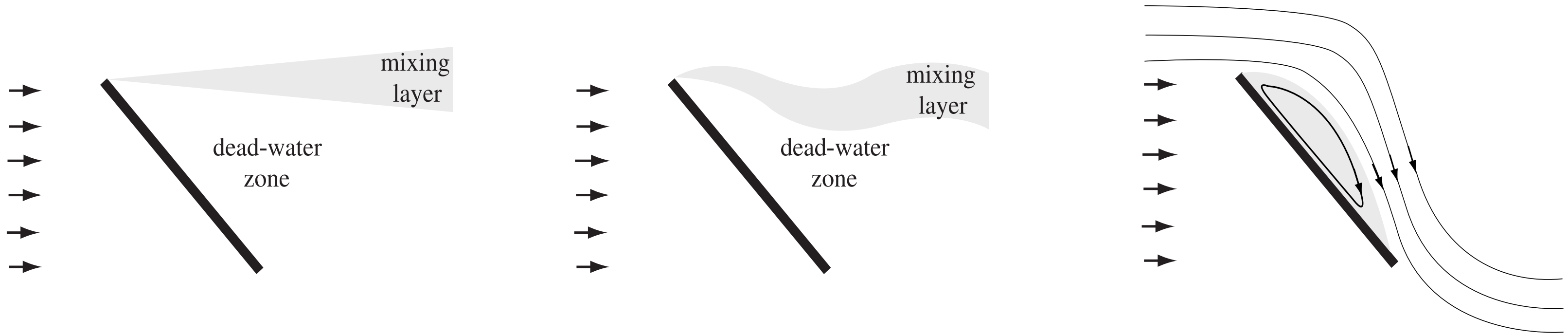
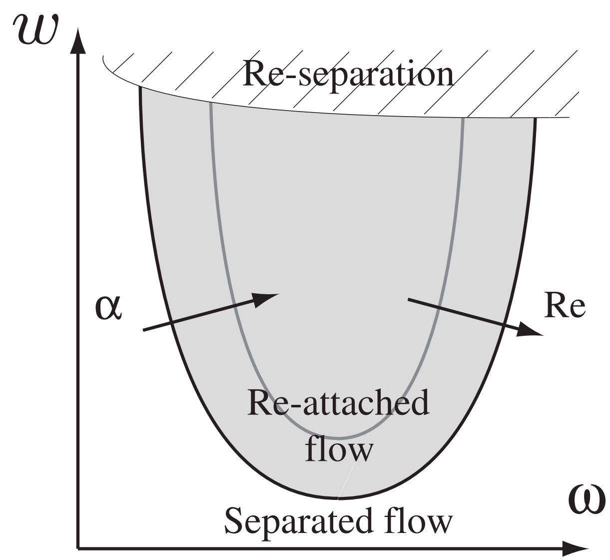


no excitation weak excitation strong excitation

- *Threshold for actuation* to achieve reattachment and *effects of amplitude w and frequency ω of actuation on bubble size x* (Nishri & Wygnanski, 1998):



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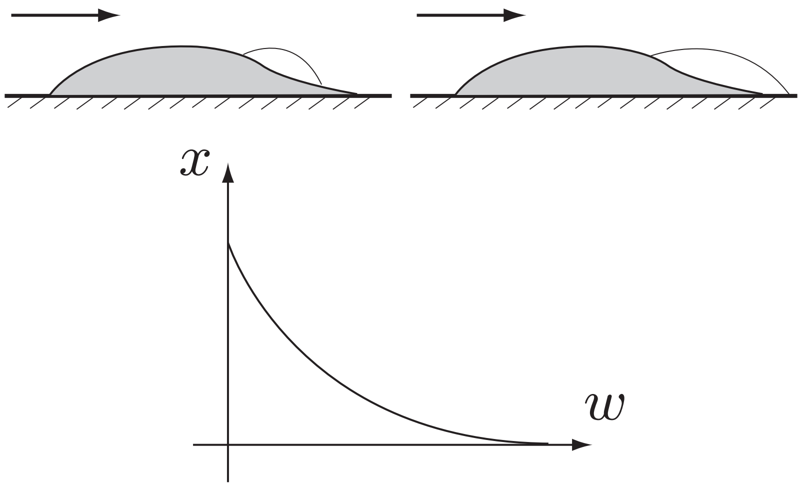

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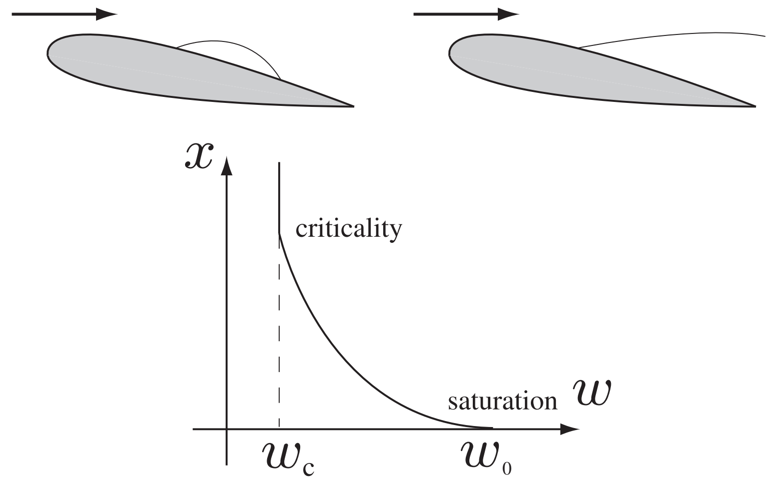

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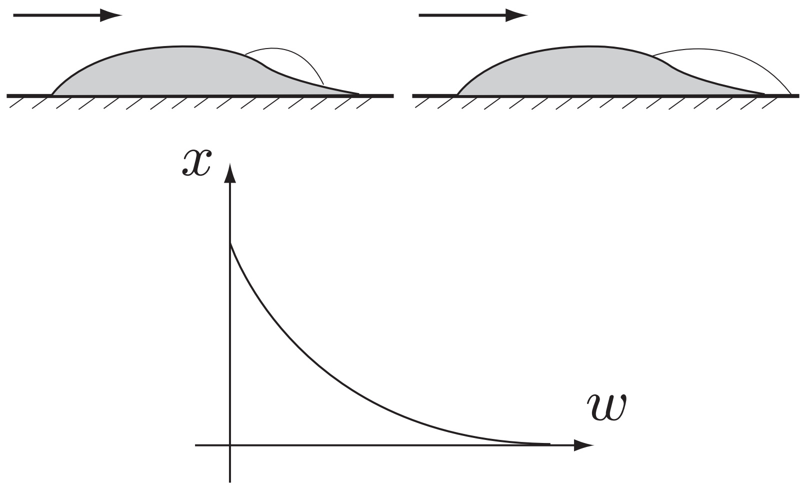

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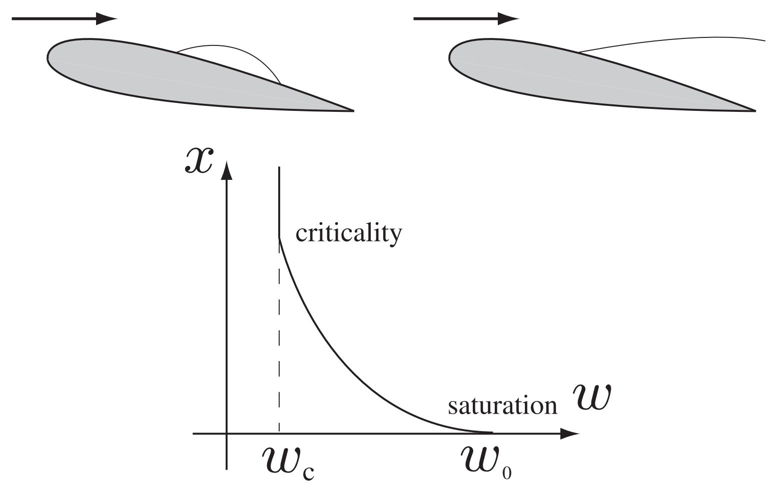

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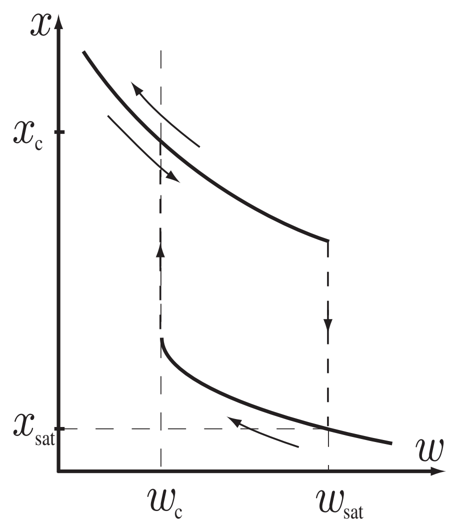

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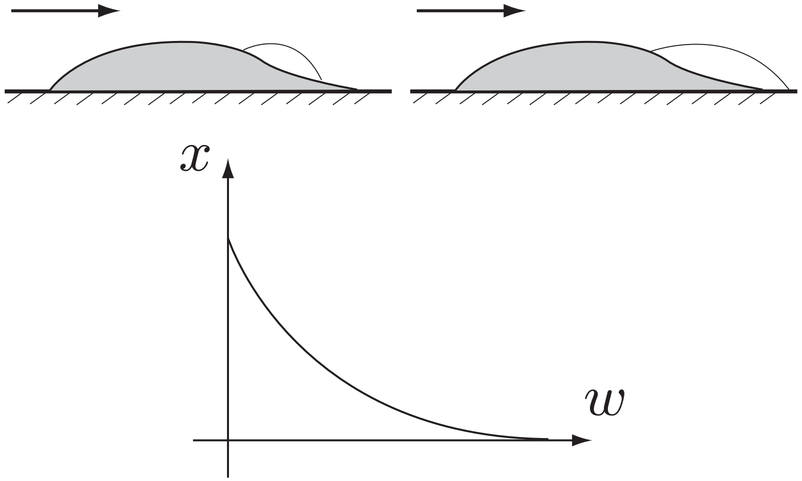

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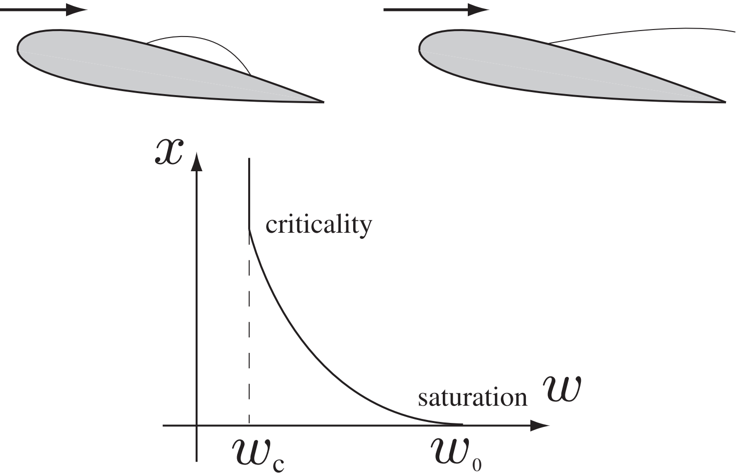

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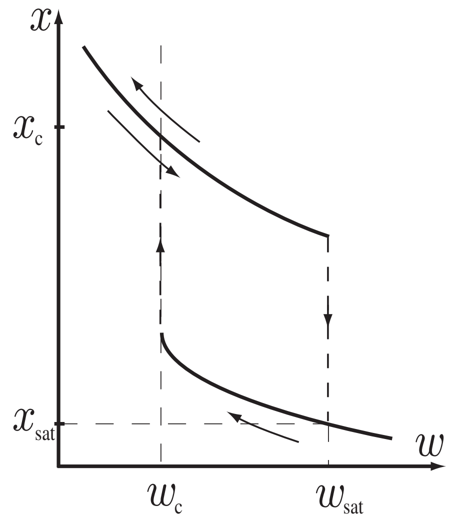
- *Conclusion*: a model should be nonlinear.



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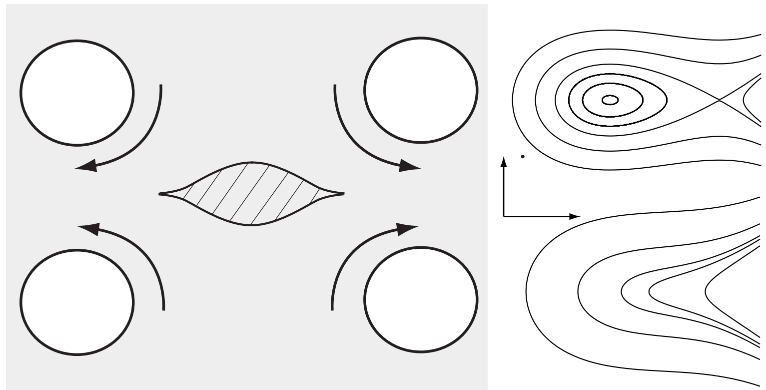

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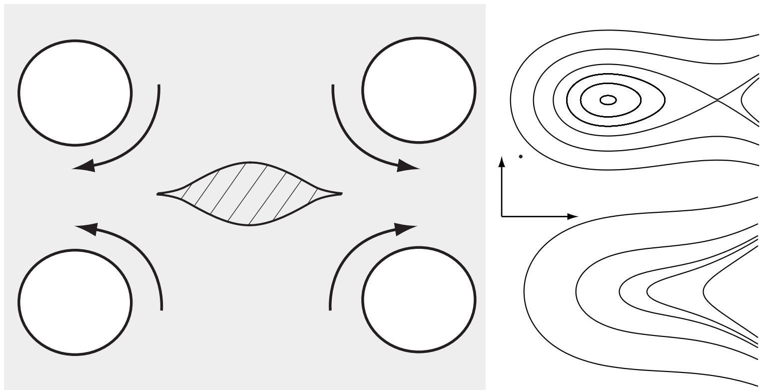

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$$\ddot{x} = -\mu\dot{x} + (ax - bx^2) + w,$$

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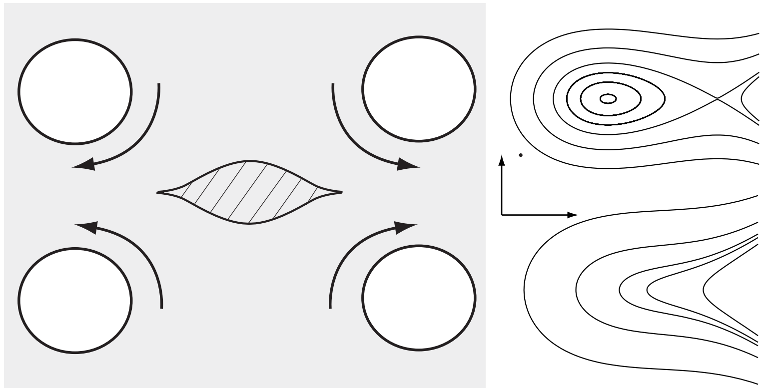

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- Bifurcation type: Takens-Bogdanov

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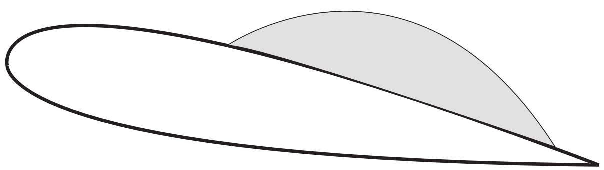

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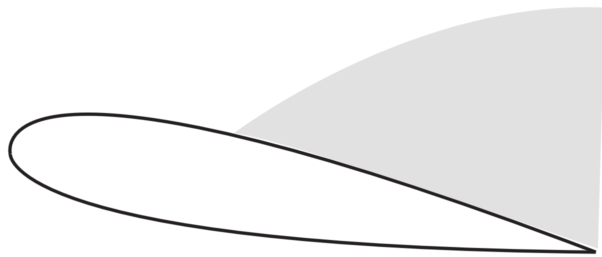

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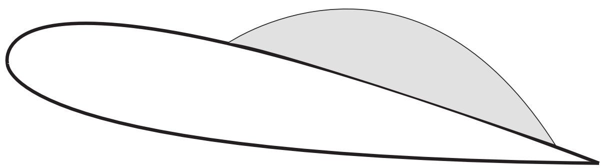

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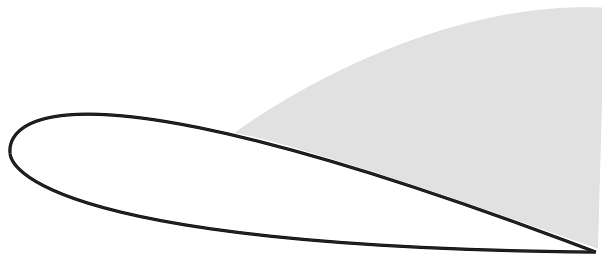

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- Naturally, the bubble size $x(t; \alpha, w)$ is a function of time t, a *flight parameter*, angle of attack α, and a *control parameter* w.

A new model: construction and analysis

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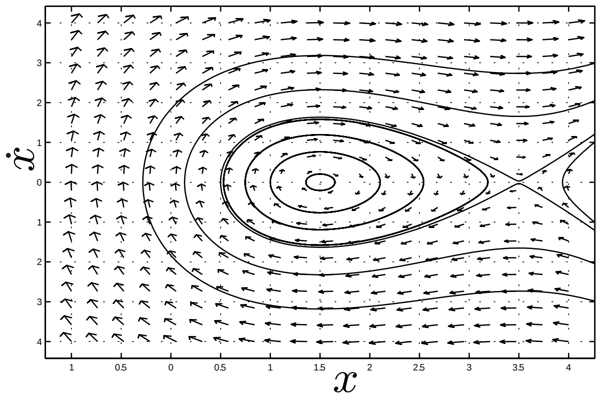
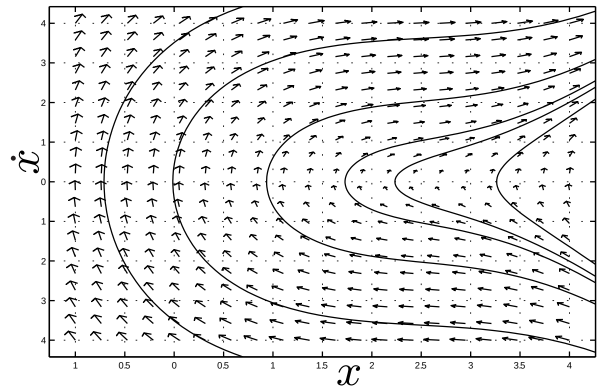
- The model is a part of the Takens-Bogdanov bifurcation:

$$\ddot{x} = -\mu \dot{x} + (x - \alpha)^2 + f(w)\,x.$$



controlled



uncontrolled

- Here $f(w) = a_1 w + a_2 w^2 + \ldots$ represents the *nonlinear response* of the separation region to actuator excitations, for instance, of a periodic form $w = w_0 \sin \omega t$. The product $f(w)\,x$ means that the *effect of actuation depends upon the bubble size* x.

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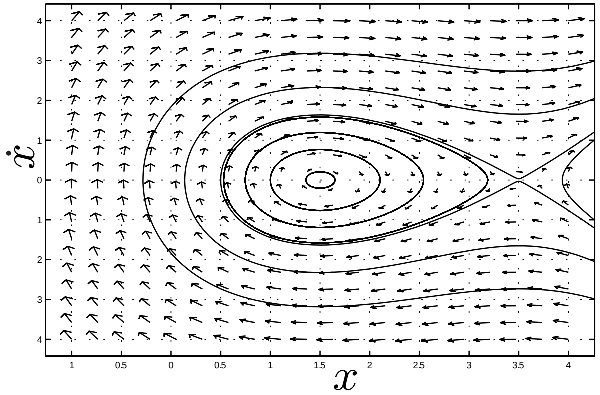

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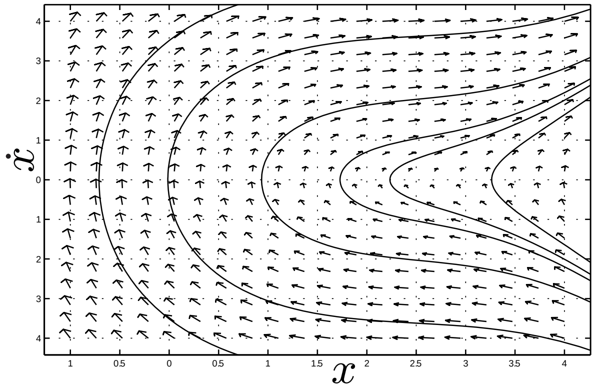

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- Prediction: separation bubble should be finite-amplitude unstable.

Hysteresis: motivation from real bubbles

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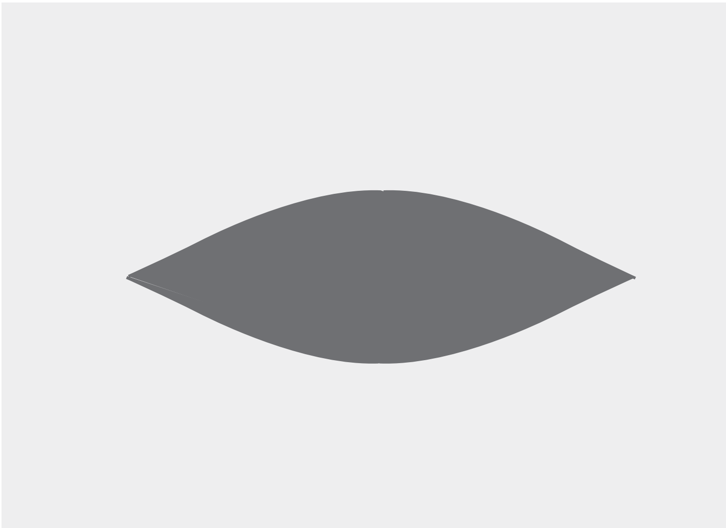
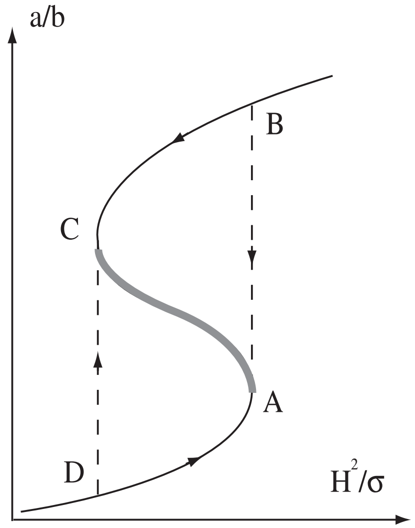
- Ferrofluid drop in a magnetic field[a]





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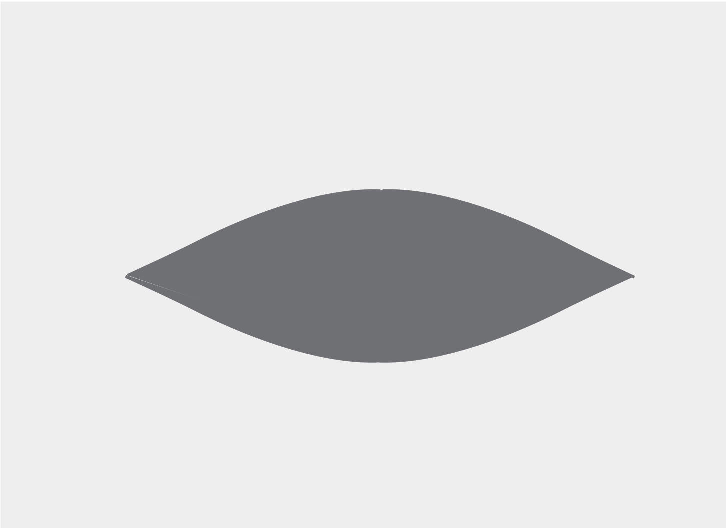

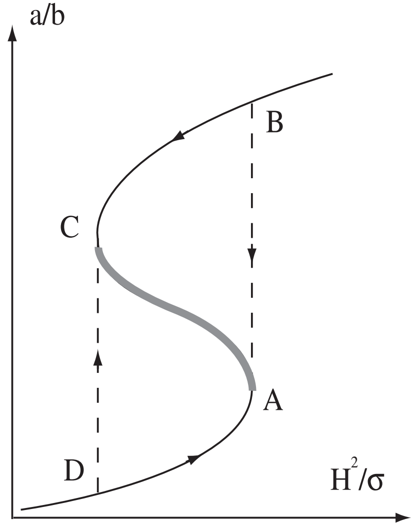

- Total energy $E_t = E_s + E_m$ is a sum of magnetic E_m and interfacial E_s contributions:

$$E_s = \sigma 2\pi a^2 e \left[e + \epsilon^{-1} \sin^{-1} \epsilon\right], \ \epsilon = \sqrt{1 - e^2}$$

$$E_m = -\frac{VH^2}{8\pi} \frac{\mu_1}{\alpha + n}, \ \alpha = \frac{\mu_1}{\mu_2 - \mu_1}.$$

- Minimizing E_t produces $H^2/\sigma = g(e)$.

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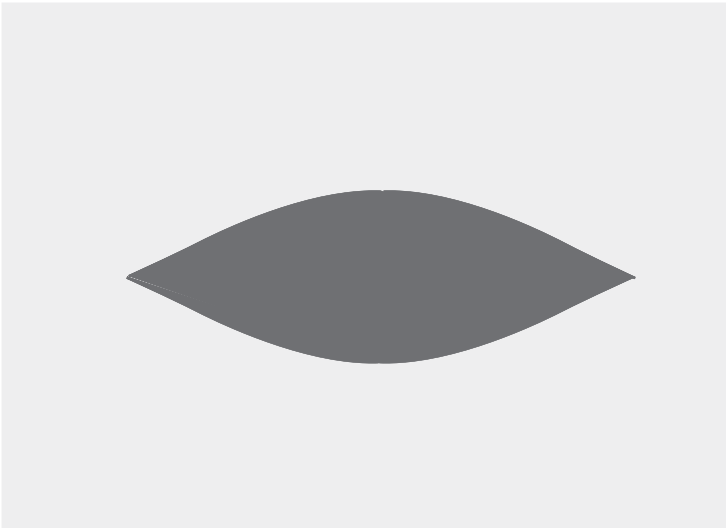
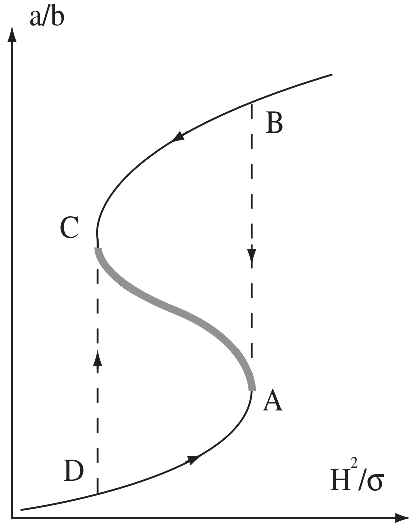
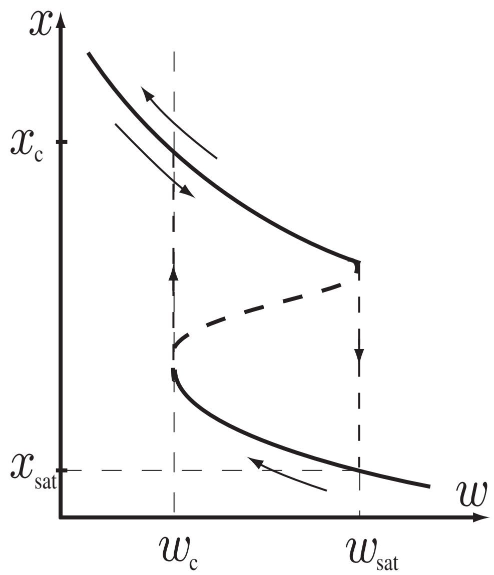




- **Conjecture**



- Total energy $E_t = E_s + E_m$ is a sum of magnetic E_m and interfacial E_s contributions:

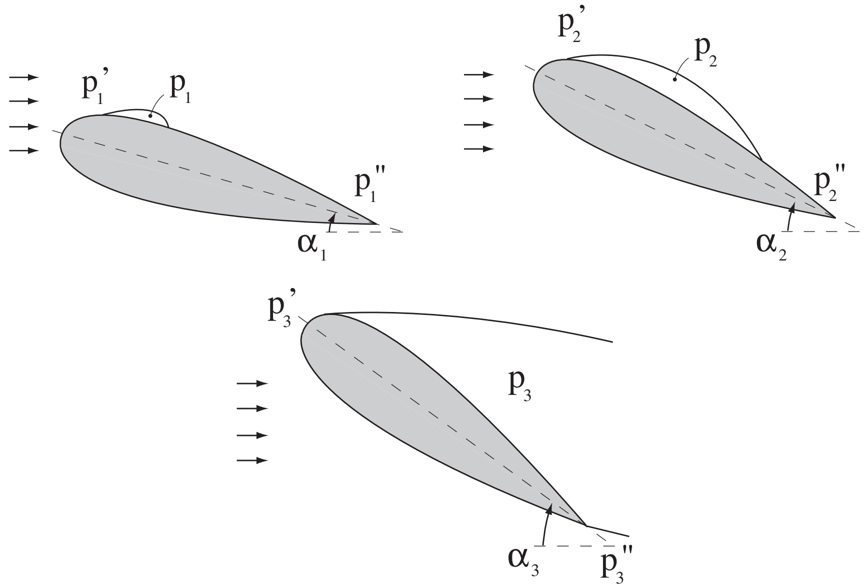
$$E_s = \sigma 2\pi a^2 e \left[e + \epsilon^{-1} \sin^{-1} \epsilon \right], \; \epsilon = \sqrt{1 - e^2}$$

$$E_m = -\frac{VH^2}{8\pi} \frac{\mu_1}{\alpha + n}, \; \alpha = \frac{\mu_1}{\mu_2 - \mu_1}.$$

- Minimizing E_t produces $H^2/\sigma = g(e)$.

Motivation: separation vs. cavitating bubble

- **Separation bubble**:

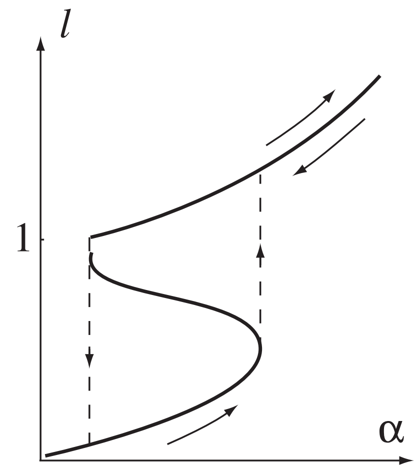


- On mechanism of separation

$$p_1' - p_1'' < p_2' - p_2'' < p_3' - p_3'',$$
$$l_1 < l_2 < l_3.$$

$$p_1 > p_2 > p_3 \text{ with } p_i < p_i',\ i = 1, 2, 3.$$

- **Cavitating bubble**:



Acosta (1955), Tulin (1953)

- The behavior of a cavitation bubble is given by for partially cavitating, $l < 1$, and supercavitating, $l > 1$, foils respectively,

$$\frac{\chi}{2\alpha} = \frac{2 - l + 2(1-l)^{1/2}}{l^{1/2}(1-l)^{1/2}},\ l < 1,$$

$$\alpha\left(\frac{2}{\chi} + 1\right) = (1-l)^{1/2},\ l > 1,$$

where α is the angle of attack, χ is a cavitation number.

Motivation: static vs. cavitating bubble

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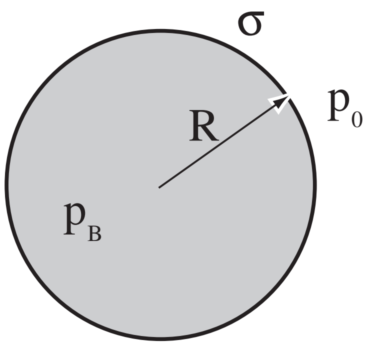
- **Static bubble**:



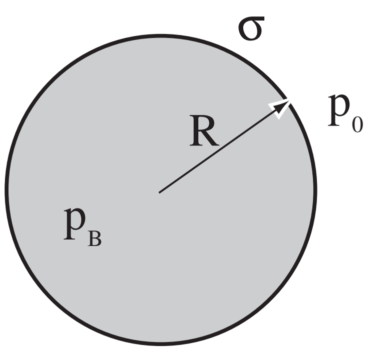
- Real static bubble behavior

$$p_B = 2\sigma/R + p_0,$$

where p_B is the pressure inside the bubble, p_0 – pressure outside the bubble, $\sigma > 0$ is the interfacial tension, and R is a radius of the bubble.

Motivation: static vs. cavitating bubble

- **Static bubble**:

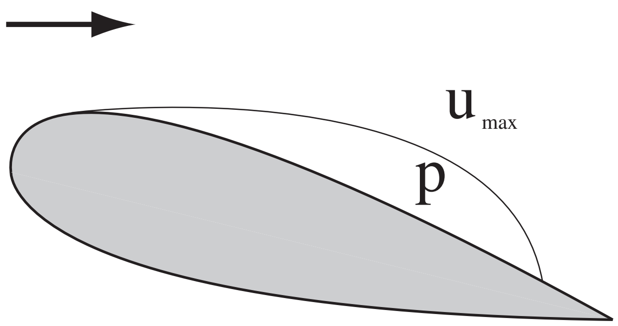


- Real static bubble behavior

$$p_B = 2\sigma/R + p_0,$$

where p_B is the pressure inside the bubble, p_0 – pressure outside the bubble, $\sigma > 0$ is the interfacial tension, and R is a radius of the bubble.

- **Cavitating hydrofoil**:



- Bubble behavior:

$$p + \frac{\rho u^2}{2} = p_{\mathrm{st}},$$

where p is a dynamic pressure, and p_{st} is the pressure of a fluid at rest (at stagnation point).

Mechanical model of hysteresis: elastic bubble

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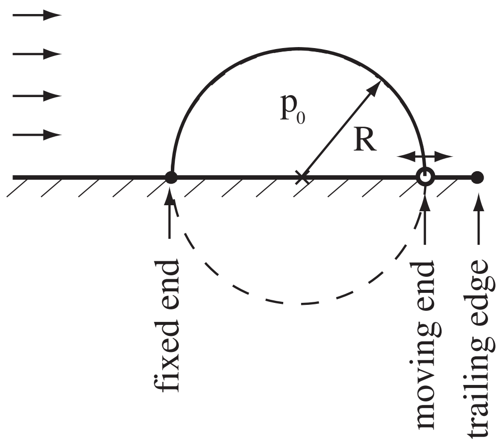
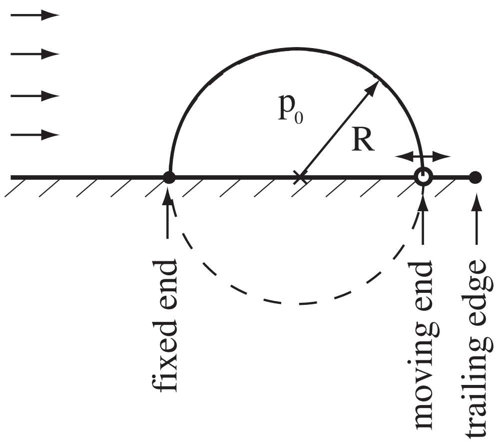
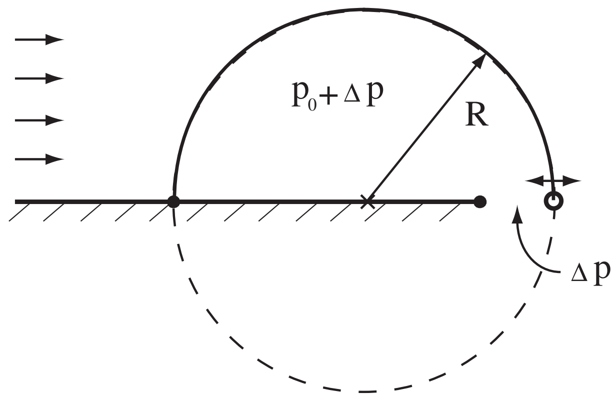
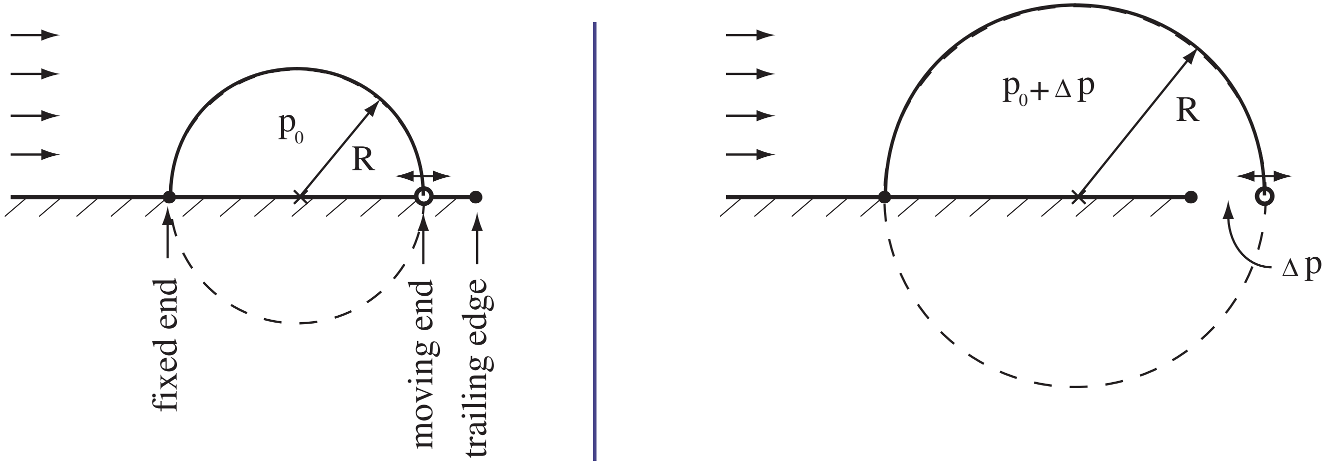


Mechanical model of hysteresis: elastic bubble





Mechanical model of hysteresis: elastic bubble



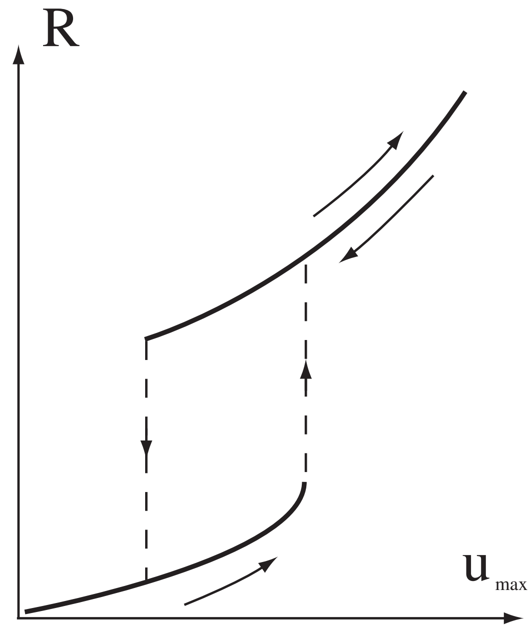
The mechanical analog of a bubble:

$$p = p_0 + \widetilde{\sigma}/R, \; p > p_0,$$

i.e. the bubble grows when the ambient pressure, $p = p_{\rm st} - \rho u_{\rm max}^2/2$, decreases.

$$u_{\rm max}^{\rm cr,>} : \;\; R_0 = \widetilde{\sigma}\left[p_{\rm st} - p_0 - \rho\left|u_{\rm max}^{\rm cr,>}\right|^2/2\right]^{-1}$$

$$u_{\rm max}^{\rm cr,<} : \;\; R_0 = \widetilde{\sigma}\left[p_{\rm st} - p_0 - \Delta p_0 - \rho\left|u_{\rm max}^{\rm cr,<}\right|^2/2\right]^{-1}$$



Conclusions

- A new physically motivated low-dimensional model of separation bubble dynamics was constructed by contrasting and appealing to similarities with actual bubble dynamics. The latter suggested
  - the proper choice of coarse variables and primary bifurcation;
  - an explaination of the nature of the hysteresis;
- Suggestions for experimental studies to improve the model:
  - investigate the finite amplitude stability of separation bubbles;
  - determine the form of the state equation for separation bubble.
- Open issues:
  - Rigorous derivation of the low-dim model by *coarsening* NSEs.
  - More close connection with experimental observations and development of a *calibration procedure*.

Acknowledgements. R.K. would like to thank Prof. Anatol Roshko for stimulating discussions.