Generalized Parametric Resonance in Electrostatically-Actuated Microelectromechanical Systems

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Parametric excitation arises quite naturally in many microelectromechanical (MEM) systems, due to the nature of electrostatic actuation. In this work we consider the response of a simple microelectromechanical (MEM) oscillator that is electrostatically driven by a non-interdigitated comb drive. The equation of motion that characterizes this system is relatively simple; it is a harmonic oscillator with a nonlinear mechanical restoring force and nonlinear electrostatic actuation and restoring forces, which due to their dependence on the oscillator's position, result in an effective parametric excitation that acts on both the linear and nonlinear terms of the equation. (This is in contrast to the case of purely linear parametric excitation, which occurs in the nonlinear Mathieu equation.) This simple, yet fundamental, difference significantly complicates the response in a manner that has been experimentally observed, yet not previously explained. Specifically, it is observed that the effective nonlinearity for the overall system depends in a qualitative manner on the amplitude of the excitation. In fact, the system has distinct effective nonlinearities for each branch of its nontrivial response, and these change in a qualitative manner as the amplitude varies. As such, the system can exhibit not only hardening and softening nonlinearities, but also mixed nonlinearities, wherein the response branches in the system's frequency response bend toward or away from one another near resonance. In addition, the interaction between the nontrivial branches can be quite intricate, and even involve global bifurcations. The presentation includes a brief derivation of the equation of motion under consideration, an outline of the analytical techniques used to reach the aforementioned results, detailed stability and bifurcation results for the system response, numerical simulations using data from an actual device, and experimental observations which clearly demonstrate the analytically predicted behavior. Relevant issues pertaining to the practical design of parametrically-excited MEM devices are also considered.